



NOVA SCHOOL OF
SCIENCE & TECHNOLOGY

COMPUTER SCIENCE DEPARTMENT

Assigning Semantics to Concurrent Programs

lecture 15 (2021-05-10)

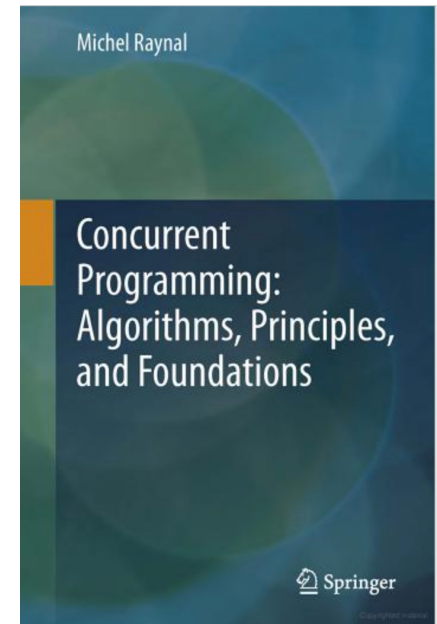
Master in Computer Science and Engineering

— Concurrency and Parallelism / 2020-21 —

João Lourenço <joao.lourenco@fct.unl.pt>

Summary

- Assigning Semantics to Concurrent Programs
 - Atomicity/Linearizability; Sequential consistency
Serializability
- Runs
 - Program state as a N-dimensional lattice
 - Consistent and Inconsistent runs
- **Reading list:**
 - **Chapter 4** of the book
Raynal M.; **Concurrent Programming:
Algorithms, Principles, and Foundations;**
Springer-Verlag Berlin Heidelberg (2013);
ISBN: 978-3-642-32026-2



Assigning Semantics to Concurrent Programs

Assigning Semantics to Concurrent Programs

$X = Y = 0$

$X = 1$
 $Y = 2$

$a = Y$
 $b = X$

$X, Y \Rightarrow$ *Global Vars*
 $a, b \Rightarrow$ *Local Vars*

Assigning Semantics to Concurrent Programs

$X = Y = 0$

$X = 1$
 $Y = 2$

$a = Y$
 $b = X$

$X, Y \Rightarrow$ Global Vars
 $a, b \Rightarrow$ Local Vars

- What are the final values for 'X', 'Y', 'a' and 'b'?

Assigning Semantics to Concurrent Programs

$X = Y = 0$

$X = 1$
 $Y = 2$

$a = Y$
 $b = X$

$X, Y \Rightarrow$ Global Vars
 $a, b \Rightarrow$ Local Vars

- What are the final values for 'X', 'Y', 'a' and 'b'?
 - $X = 1, Y = 2$

Assigning Semantics to Concurrent Programs

$X = Y = 0$

$X = 1$
 $Y = 2$

$a = Y$
 $b = X$

$X, Y \Rightarrow$ Global Vars
 $a, b \Rightarrow$ Local Vars

- What are the final values for 'X', 'Y', 'a' and 'b'?
 - $X = 1, Y = 2, a = ?, b = ?$

Assigning Semantics to Concurrent Programs

$X = Y = 0$

$X = 1$
 $Y = 2$

$a = Y$
 $b = X$

$X, Y \Rightarrow$ Global Vars
 $a, b \Rightarrow$ Local Vars

- What are the final values for 'X', 'Y', 'a' and 'b'?
 - $X = 1, Y = 2, a = ?, b = ?$
- Depends on the interleavings of the statements
 - Sequential Consistency [Lamport'79]
 - Program behavior = set of interleavings

Assigning Semantics to Concurrent Programs

$X = Y = 0$

$X, Y \Rightarrow$ Global Vars
 $a, b \Rightarrow$ Local Vars

$X = 1$
 $Y = 2$

$a = Y$
 $b = X$

$X = 1$

$Y = 2$

$a = Y$

$b = X$

Assigning Semantics to Concurrent Programs

$X = Y = 0$

$X, Y \Rightarrow$ Global Vars
 $a, b \Rightarrow$ Local Vars

$X = 1$
 $Y = 2$

$a = Y$
 $b = X$

$X = 1$

$Y = 2$

$a = Y$

$b = X$

$a=2, b=1$

Assigning Semantics to Concurrent Programs

$X = Y = 0$

$X, Y \Rightarrow$ Global Vars
 $a, b \Rightarrow$ Local Vars

$X = 1$
 $Y = 2$

$a = Y$
 $b = X$

$X = 1$

$X = 1$

$Y = 2$

$a = Y$

$a = Y$

$b = X$

$b = X$

$Y = 2$

$a=2, b=1$

Assigning Semantics to Concurrent Programs

$X = Y = 0$

$X, Y \Rightarrow$ Global Vars
 $a, b \Rightarrow$ Local Vars

$X = 1$
 $Y = 2$

$a = Y$
 $b = X$

$X = 1$

$Y = 2$

$a = Y$

$b = X$

$a=2, b=1$

$X = 1$

$a = Y$

$b = X$

$Y = 2$

$a=0, b=1$

Assigning Semantics to Concurrent Programs

$X = Y = 0$

$X, Y \Rightarrow$ Global Vars
 $a, b \Rightarrow$ Local Vars

$X = 1$
 $Y = 2$

$a = Y$
 $b = X$

$X = 1$

$Y = 2$

$a = Y$

$b = X$

$a=2, b=1$

$X = 1$

$a = Y$

$b = X$

$Y = 2$

$a=0, b=1$

$X = 1$

$a = Y$

$Y = 2$

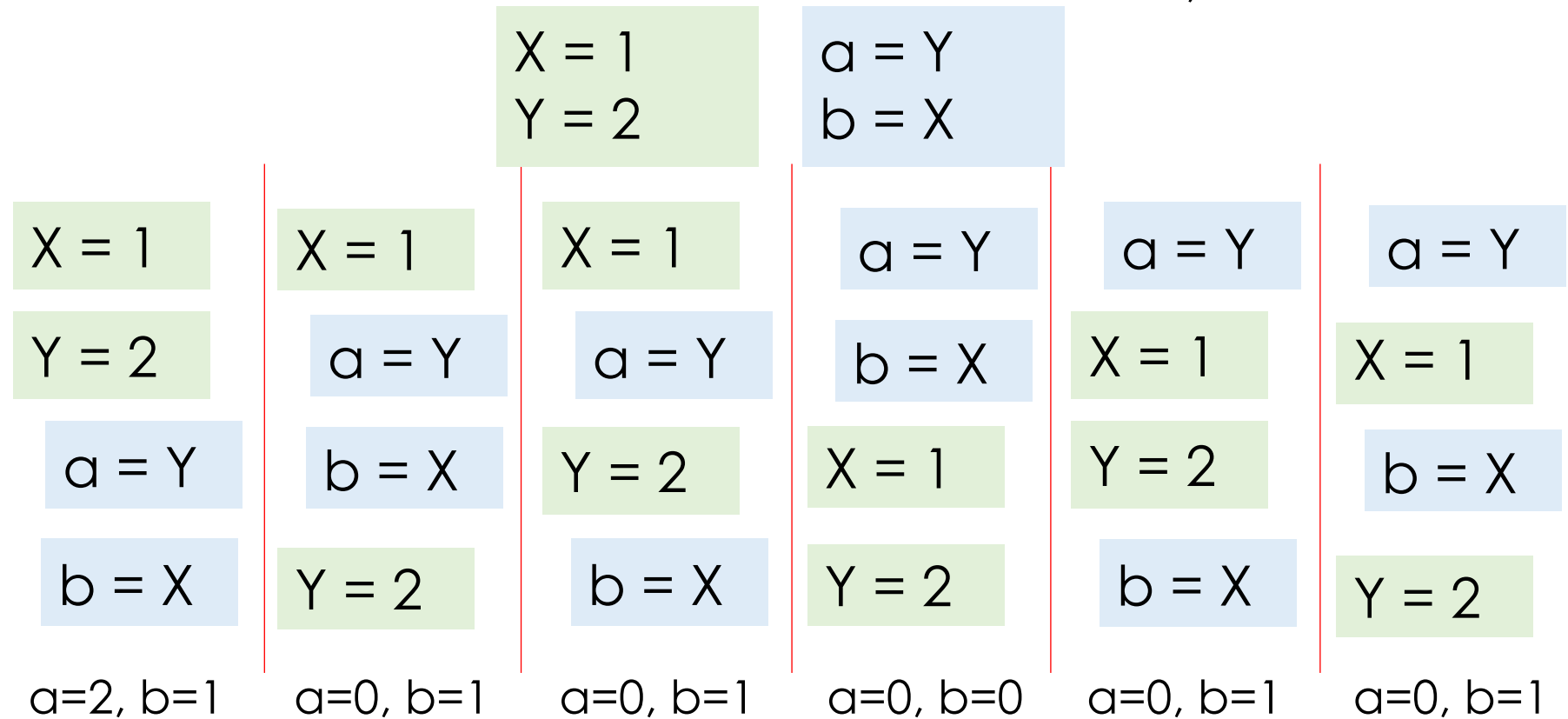
$b = X$

$a=0, b=1$

Assigning Semantics to Concurrent Programs

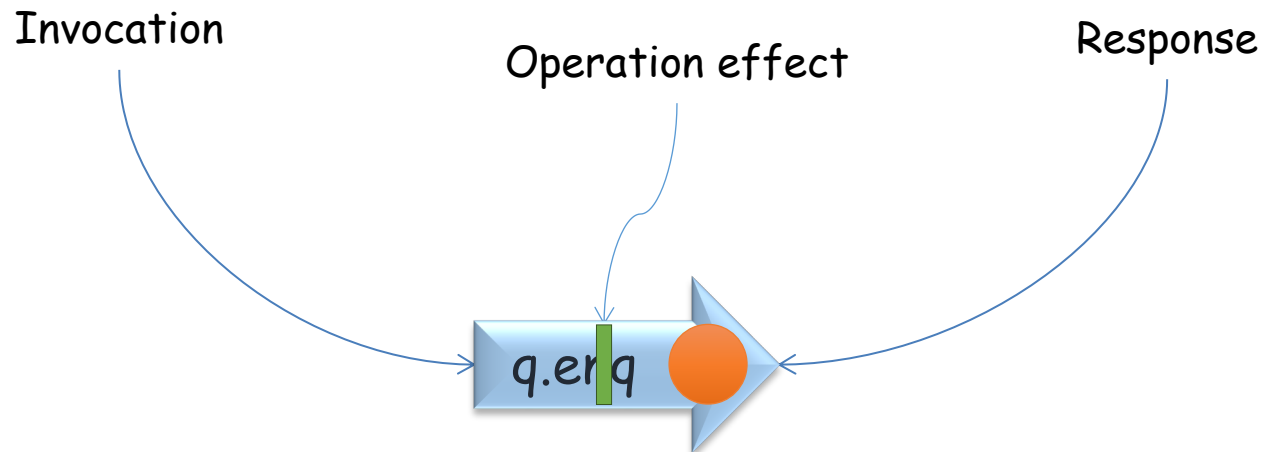
$X = Y = 0$

$X, Y \Rightarrow$ Global Vars
 $a, b \Rightarrow$ Local Vars



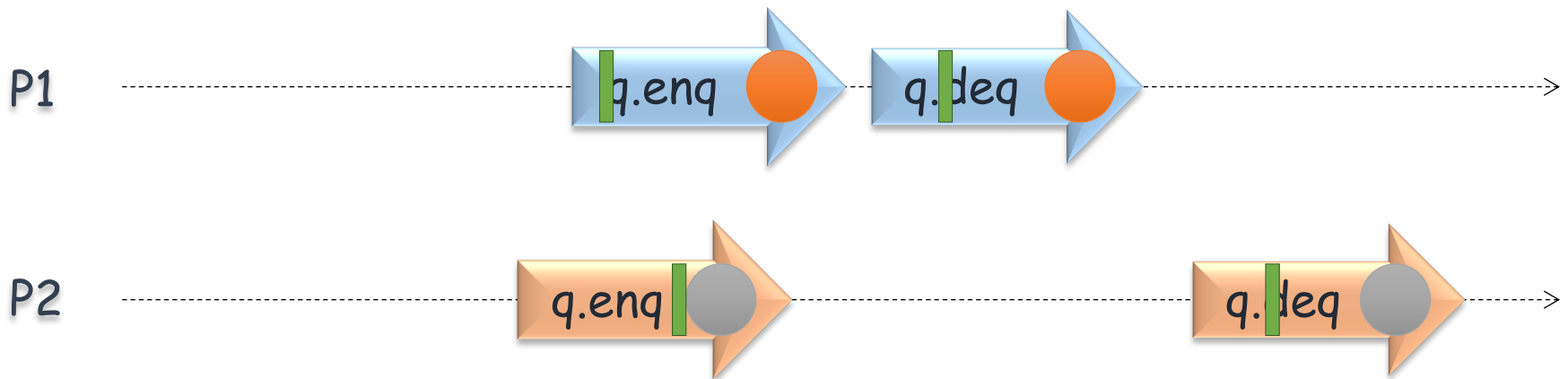
Sequential Consistency and Linearizability

- Each operation should appear to “take effect” instantaneously at some moment between its invocation and response



Sequential Consistency and Linearizability

- Each operation should appear to “take effect” instantaneously at some moment between its invocation and response
- Object is correct if its behaviour can be explained by a sequential execution of the (concurrent) operations



Sequential Consistency

- Instructions are executed by the order they appear in the program
- Memory behaves as a shared array
 - Reads and writes are effective immediately
- Be aware that:
 - This is naturally true for sequential programs...
 - But it is not true for concurrent programs!

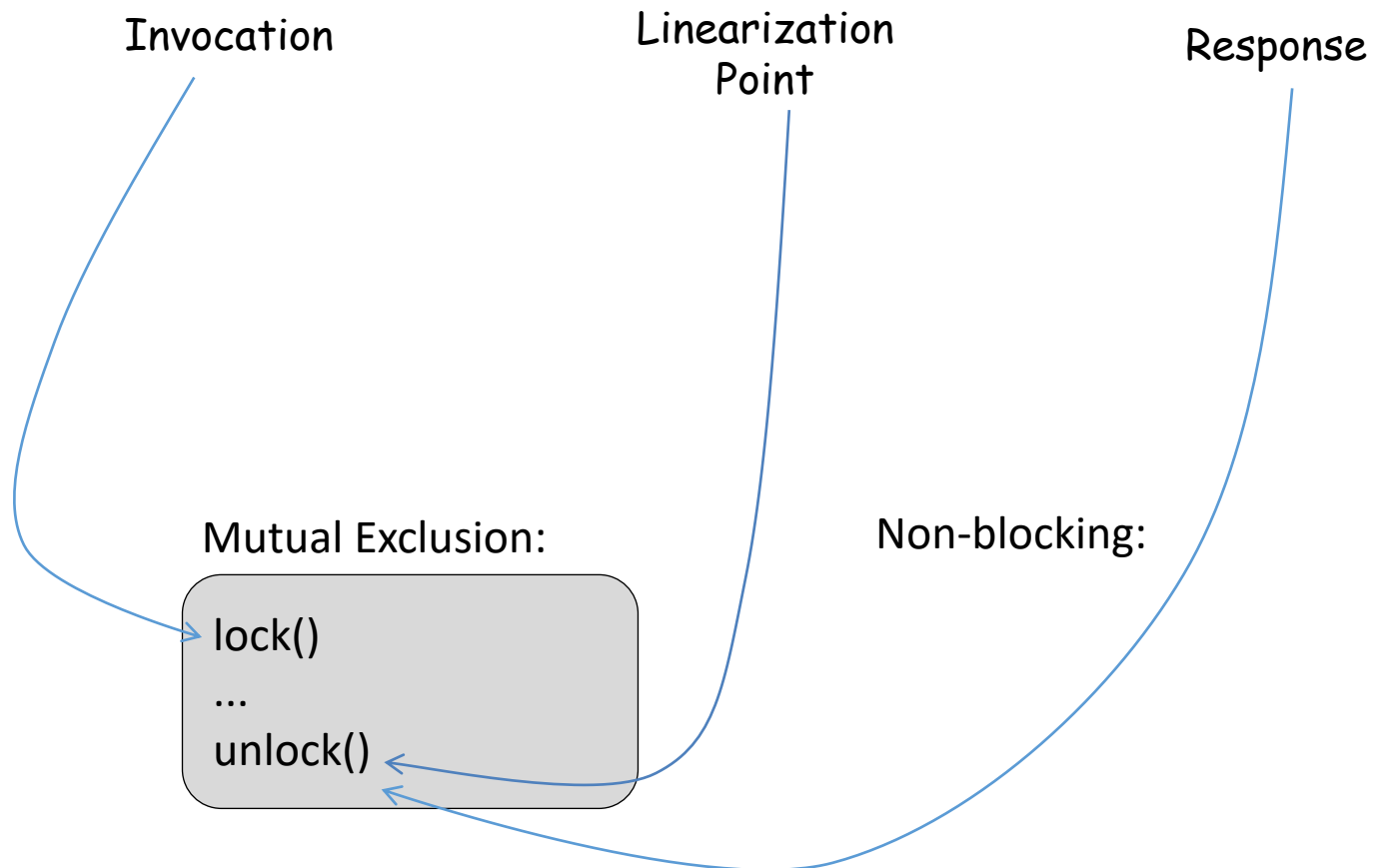
Linearizability

- Instructions are executed by the order they appear in the program
- Memory behaves as a shared array
 - Reads and writes are effective immediately
- Need to preserve real time order
- Be aware that:
 - This is naturally true for sequential programs...
 - But it is not true for concurrent programs!

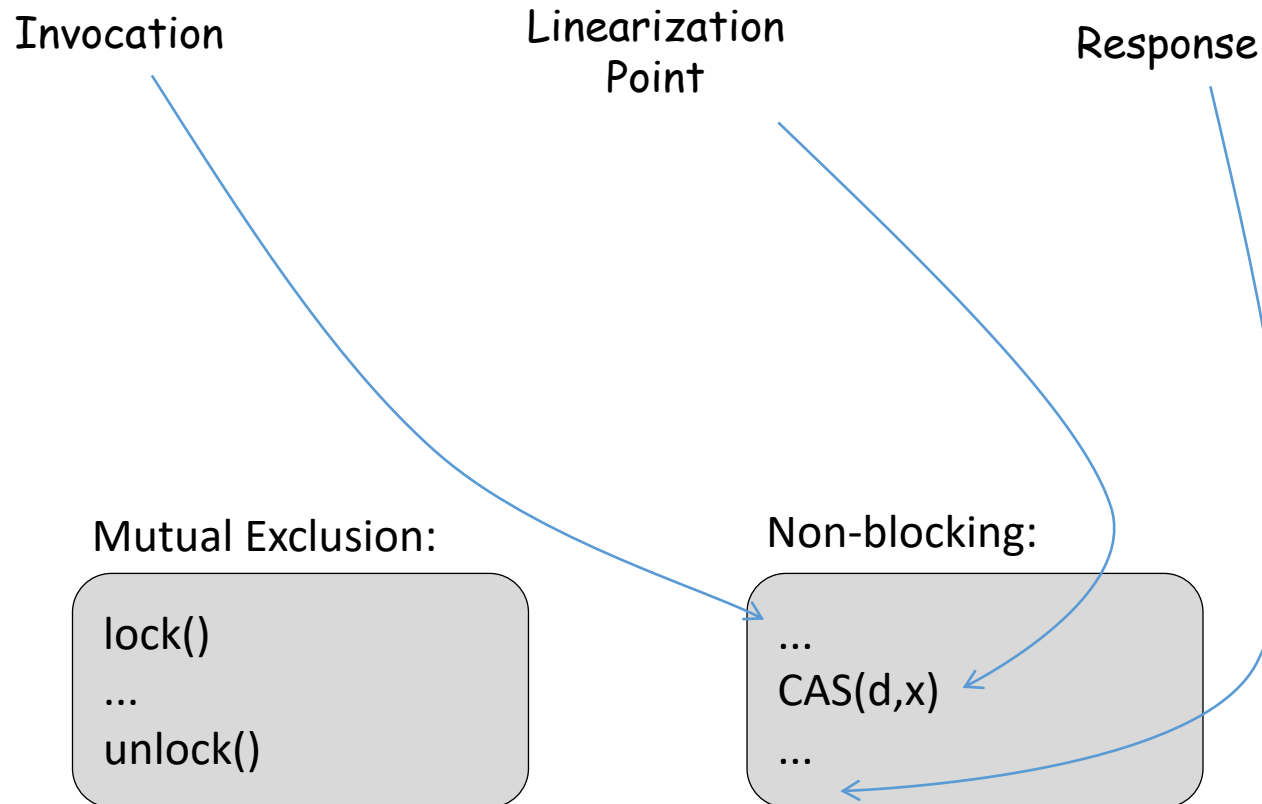
Linearizability

- Permits programmers to specify and reason about concurrent objects using known techniques from the sequential domain
- Provides the illusion that each operation applied by concurrent processes takes effect instantaneously between its invocation and its response

Terminology

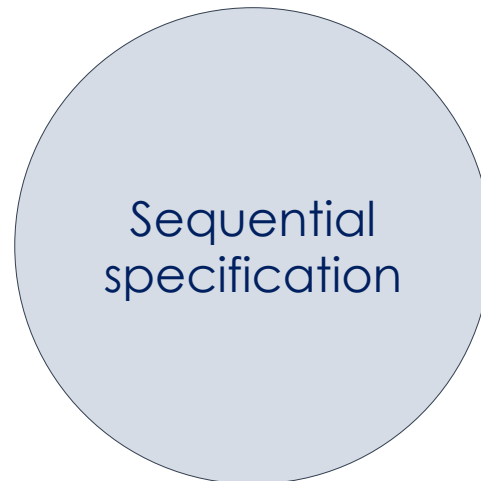


Terminology

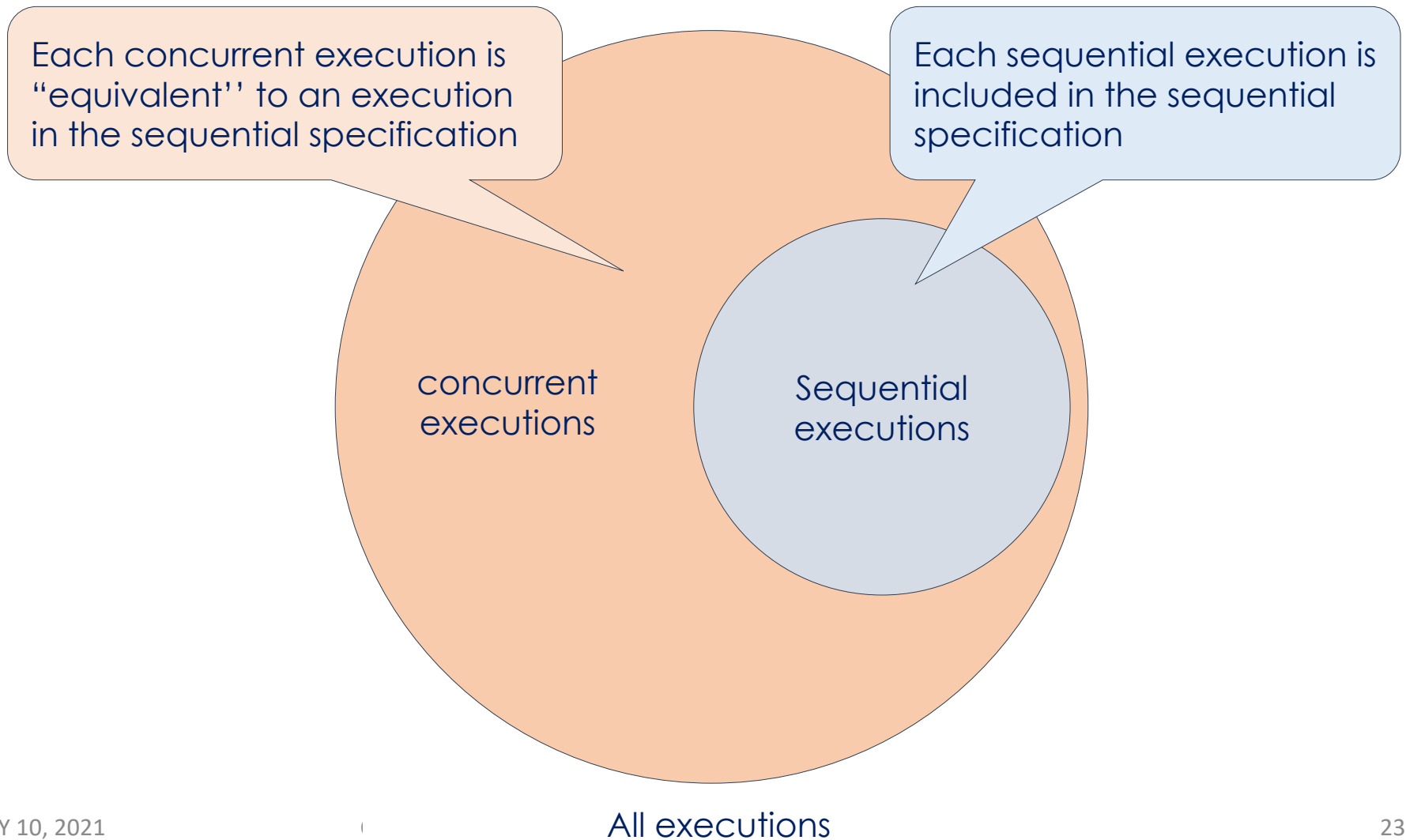


Sequential Specification of an object

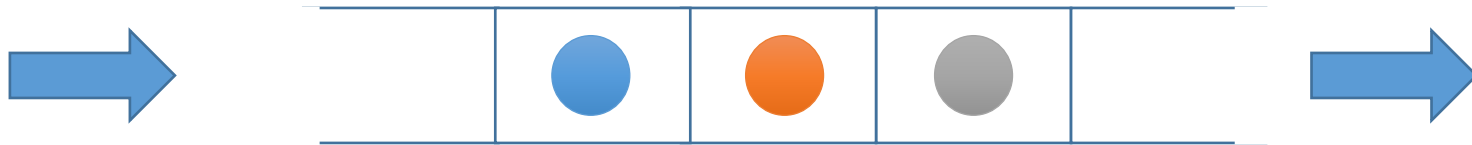
- **Queue:** In sequential executions, the enqueue operation inserts a value to the queue and the dequeue operation returns and deletes the oldest value in the queue



Linearizability for an object



Example: Concurrent Queue

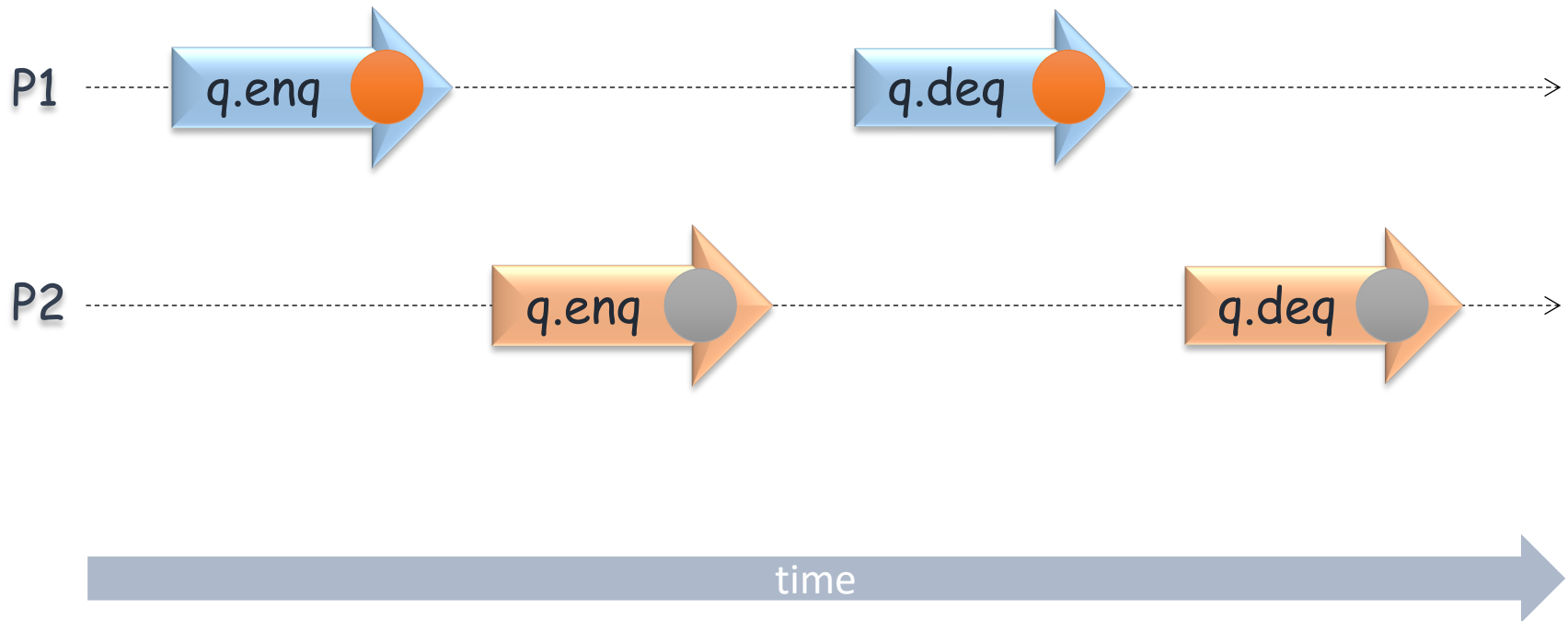


`q.enq(●)`

`q.deq(●)`

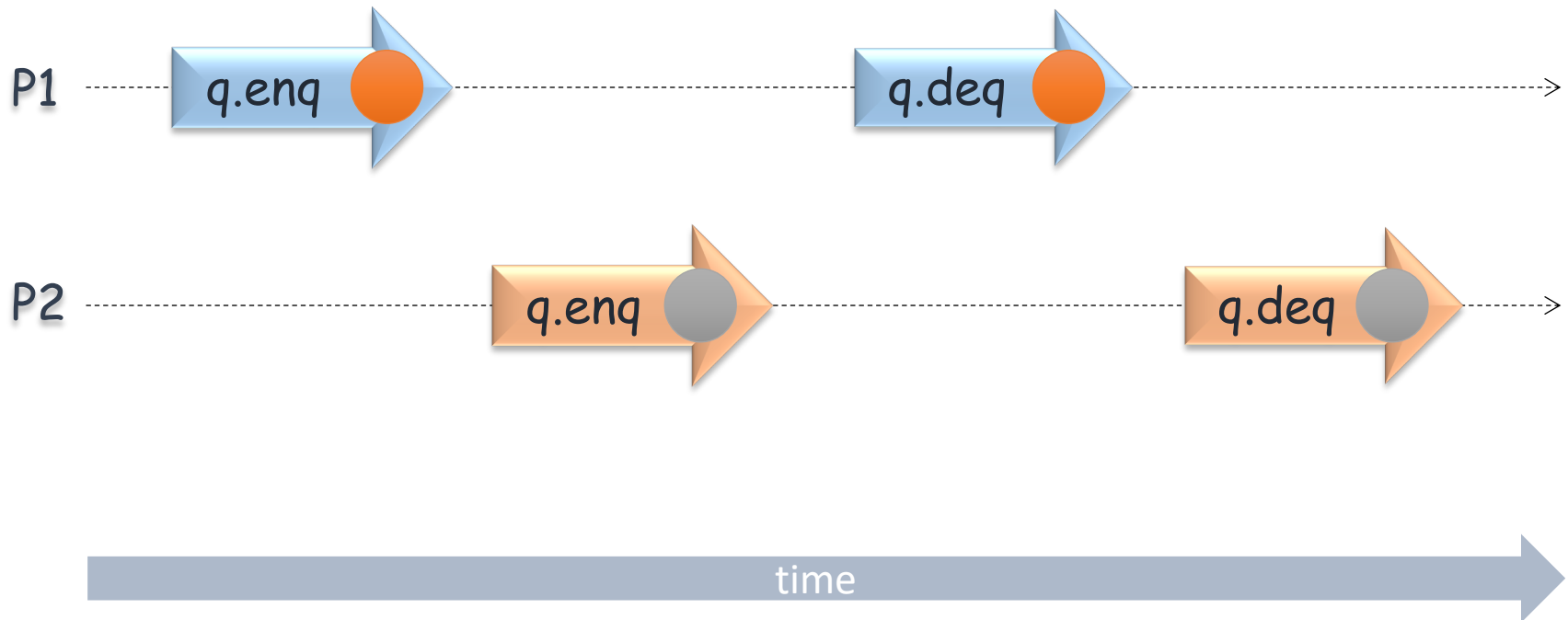
Sequential Execution

Is this execution	reasonable?	Yes	
-------------------	-------------	-----	--



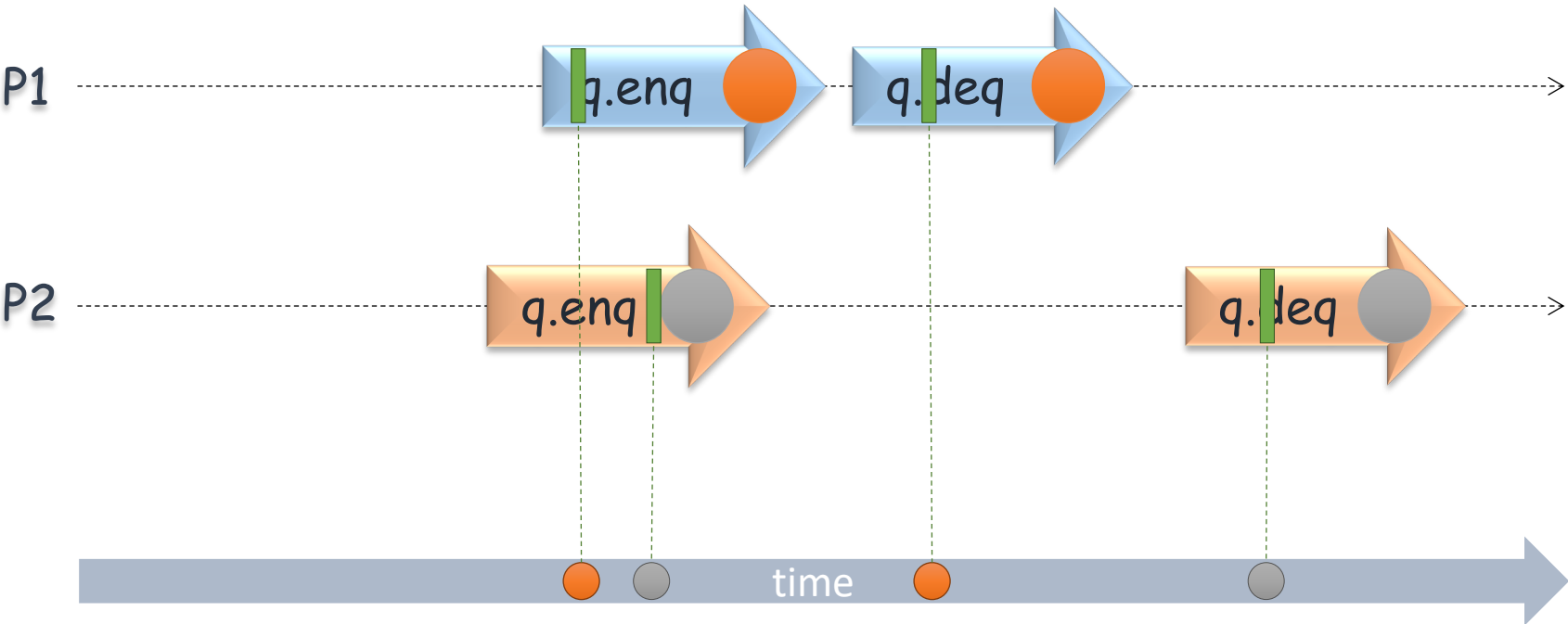
Concurrent Execution

Is this execution reasonable? ???



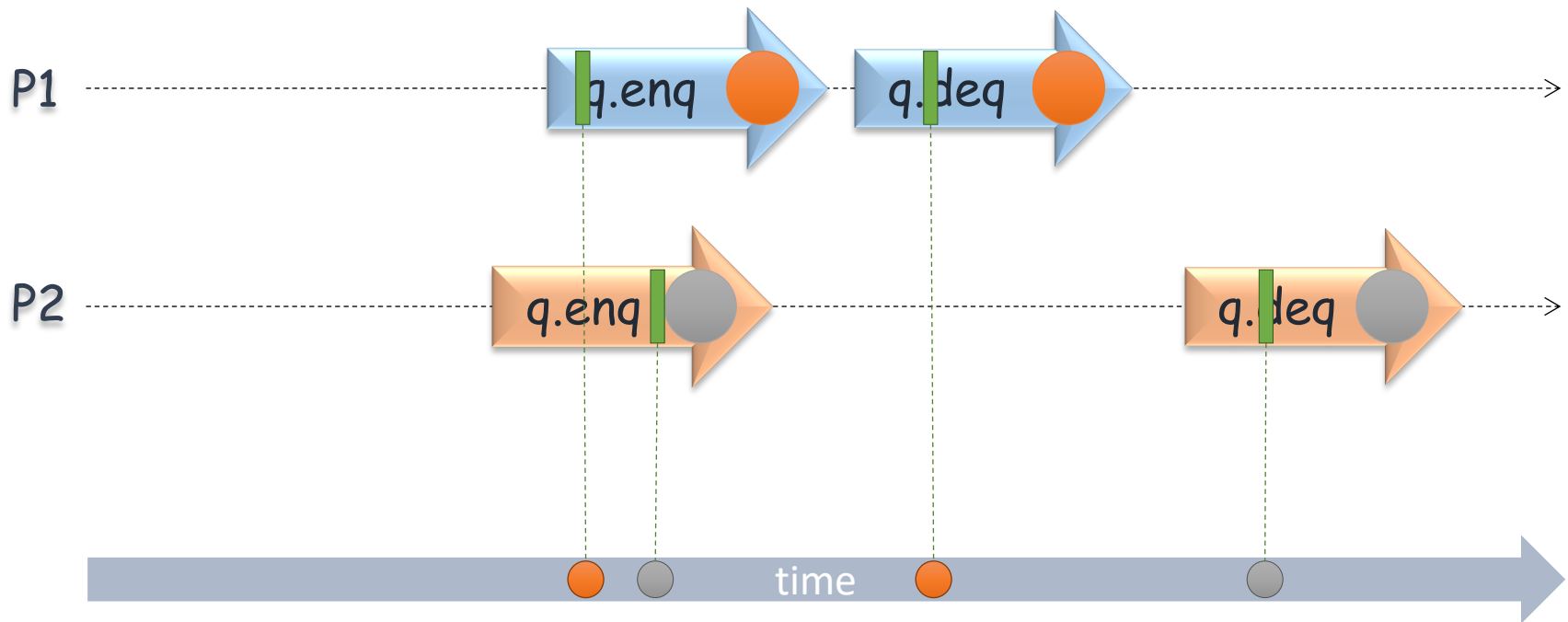
Concurrent Execution

Is this execution reasonable? **Yes**



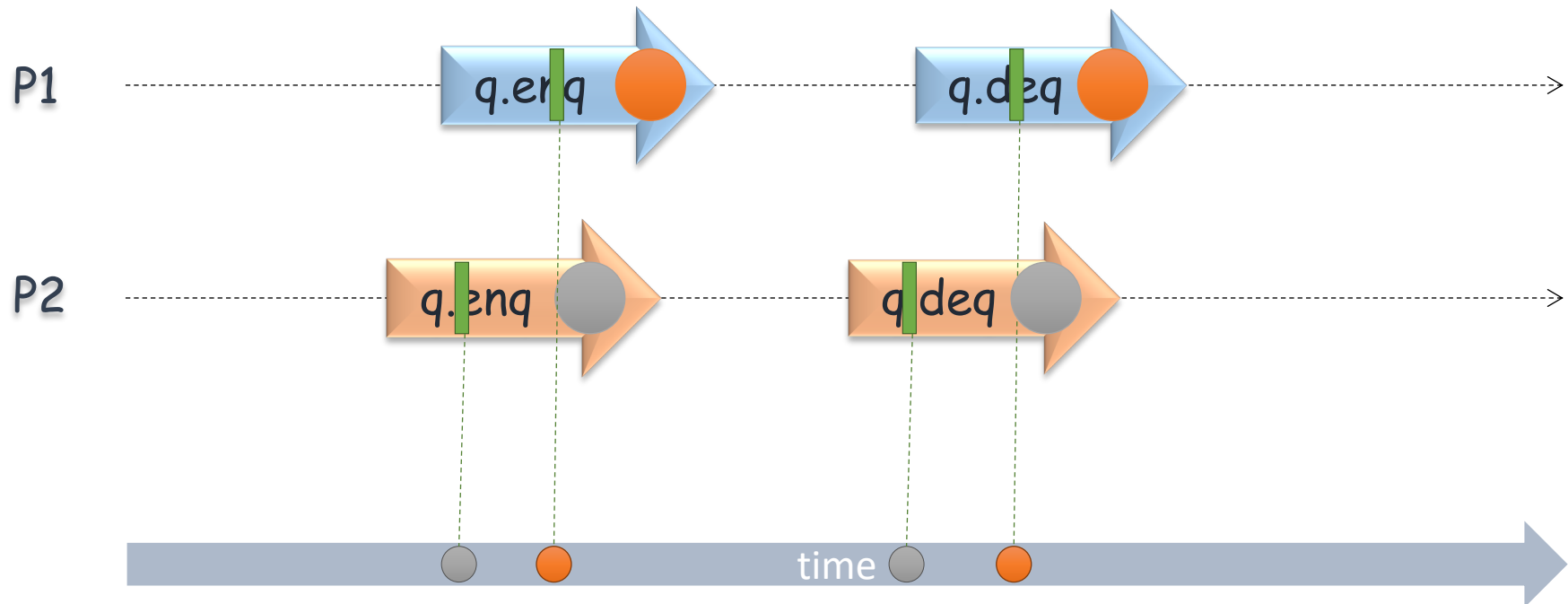
Concurrent Execution

Is this execution reasonable? **Yes**



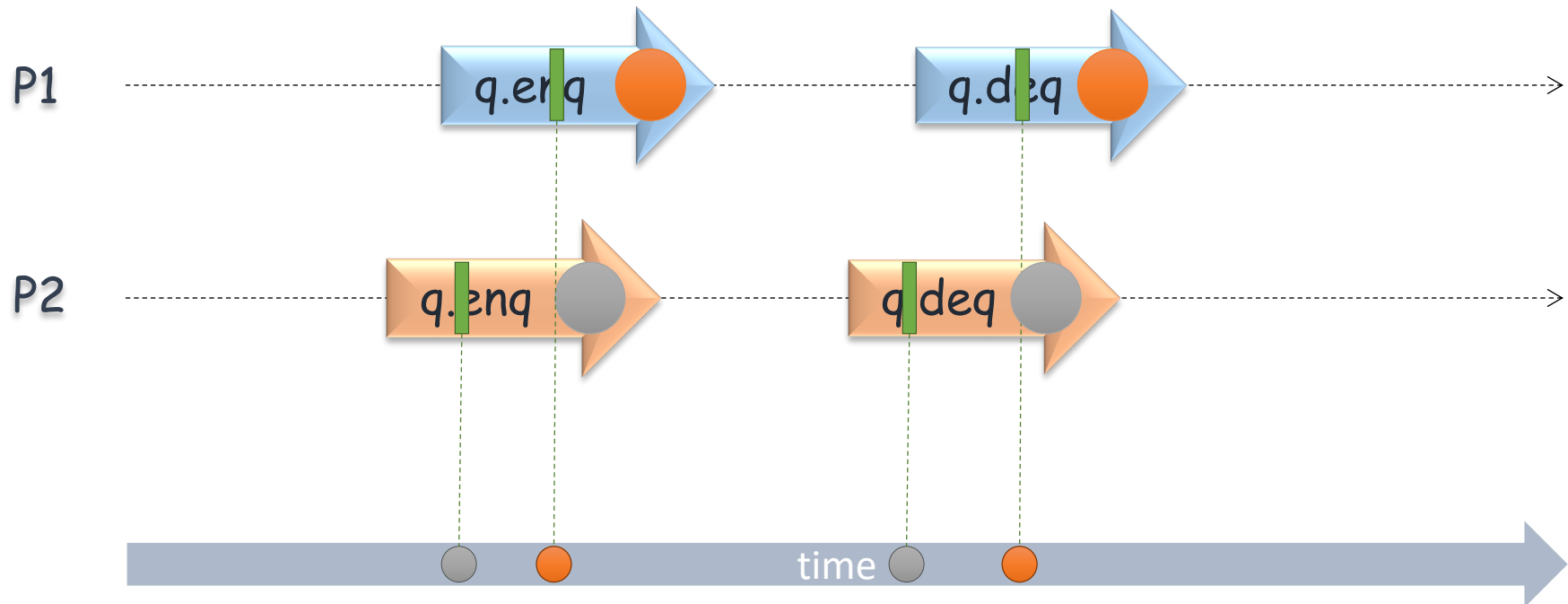
Linearizability

Is this execution **linearizable**? **Yes**



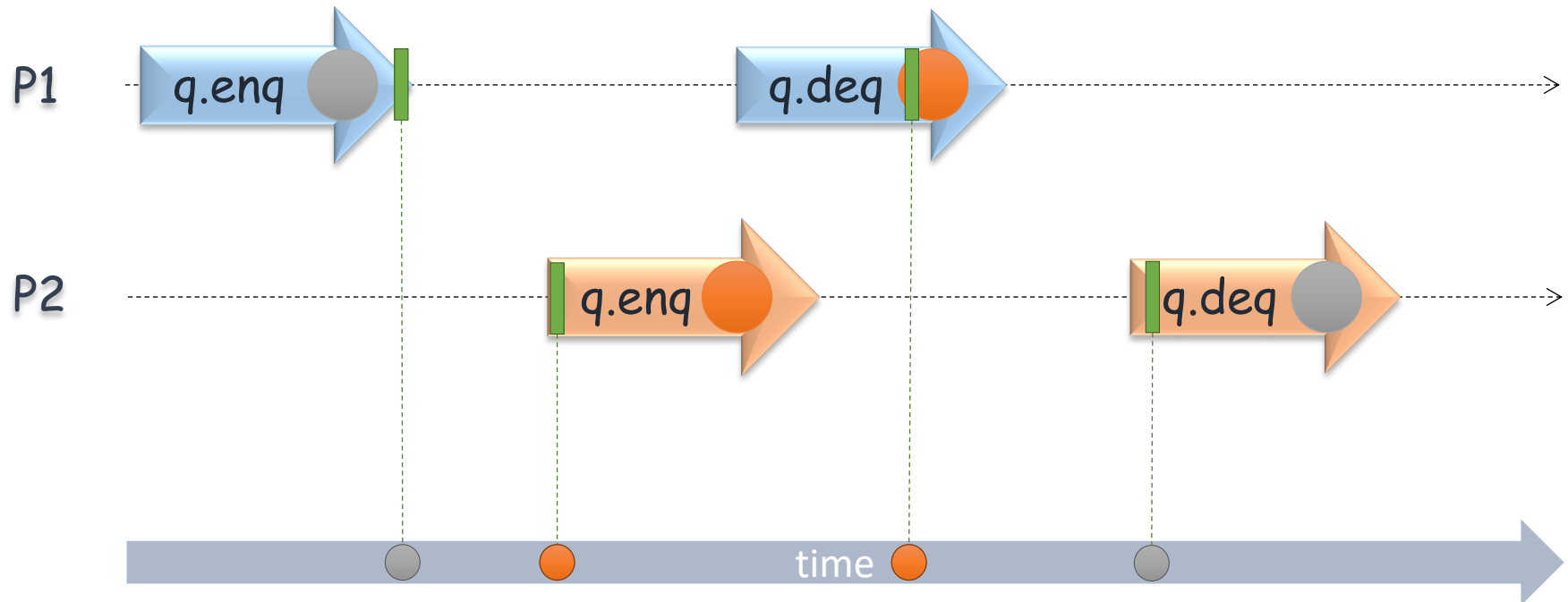
Linearizability

Is this execution **linearizable**? **Yes**



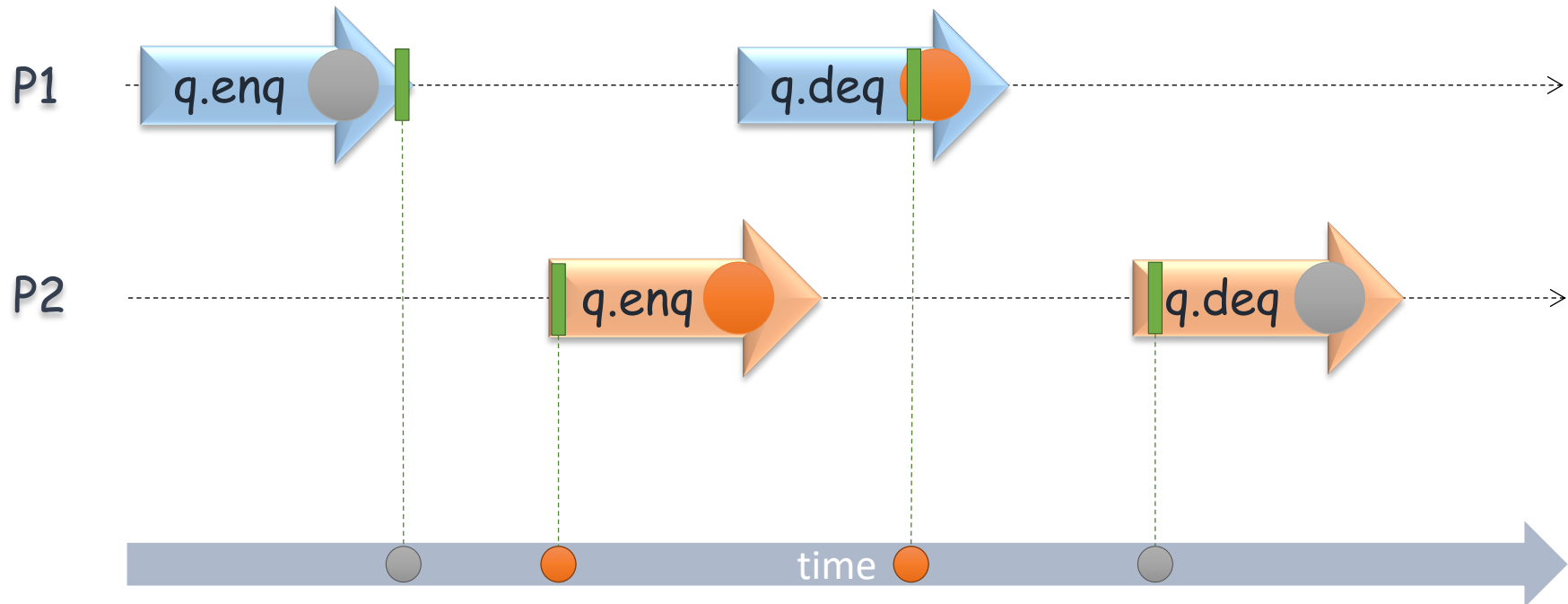
Linearizability

Is this execution **linearizable**? No, BUT ...



Linearizability

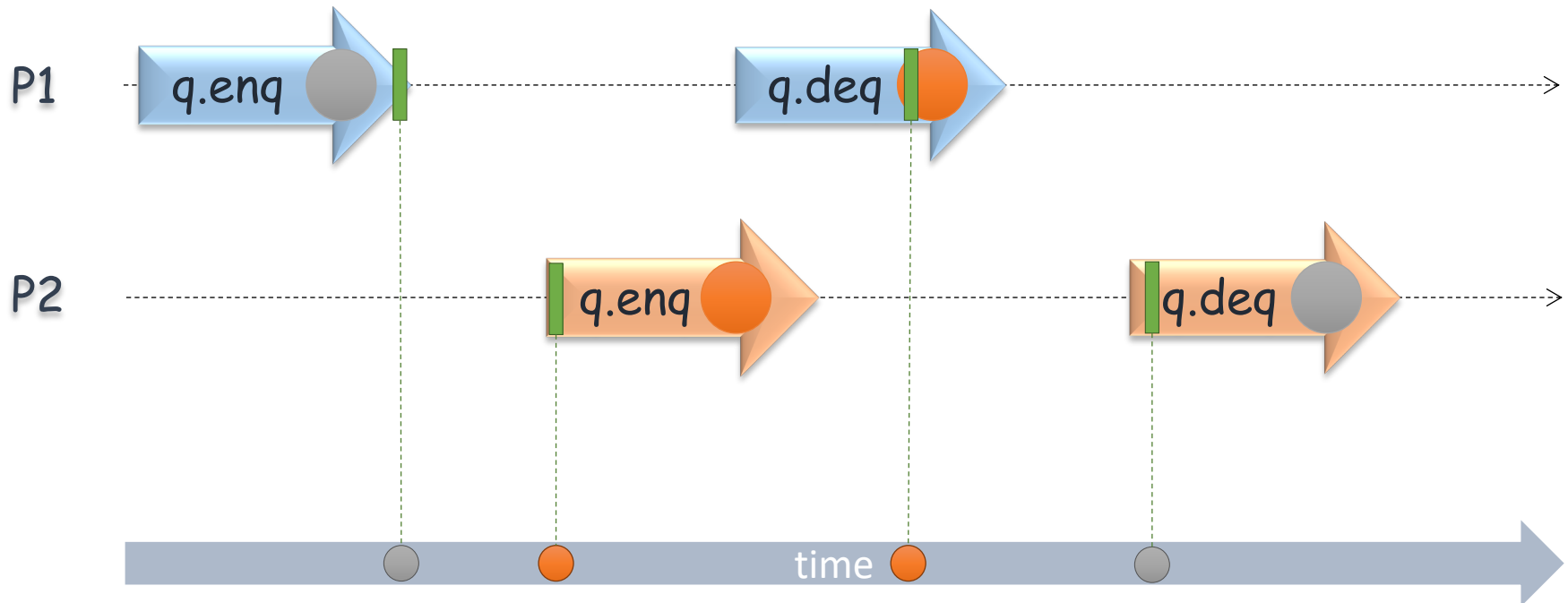
Is this execution **linearizable**? No, BUT ...



~~Linearizability~~

Sequential Consistency

Is this execution **linearizable**? No, BUT ...

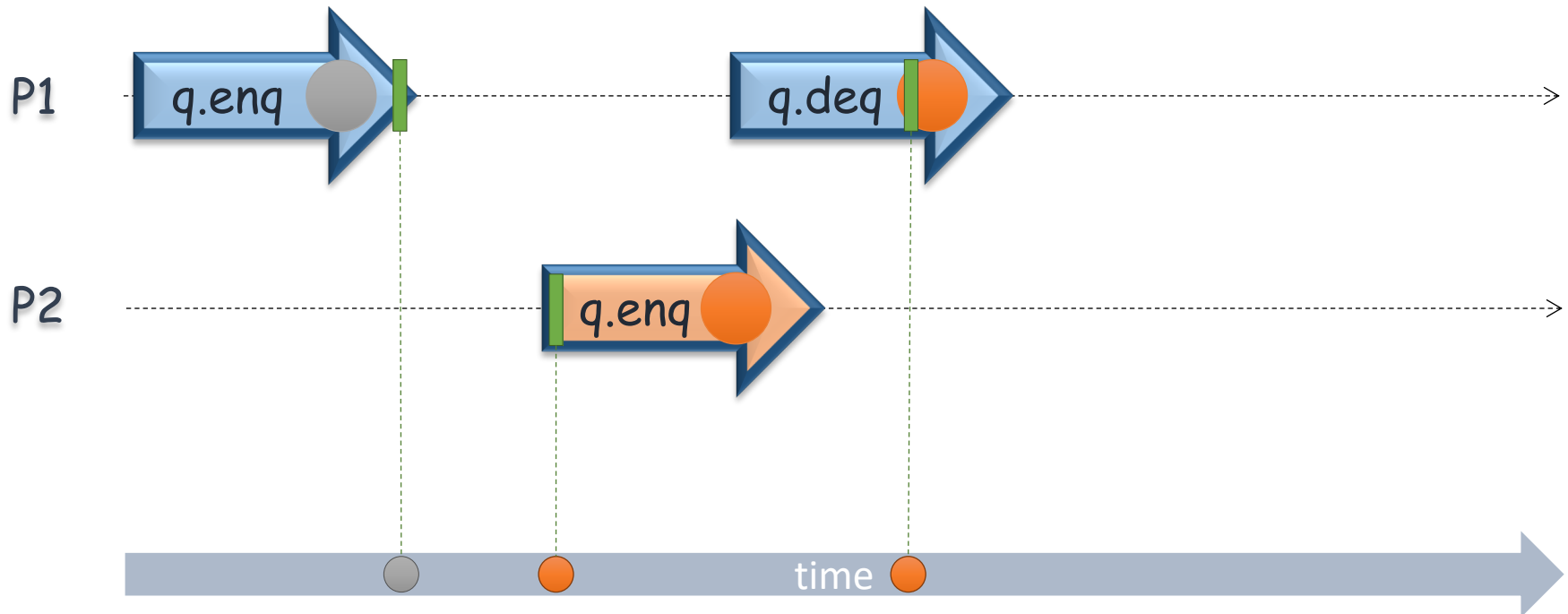


Sequential Consistency

- No need to preserve real time order
 - Operations can “slide” within the same thread but not be reordered
 - Parallel operations can be reordered as convenient
- Each operation should “take effect” instantaneously
- The result of the concurrent execution of a program P must be equivalent to some sequential execution of the same program P

Sequential Consistency

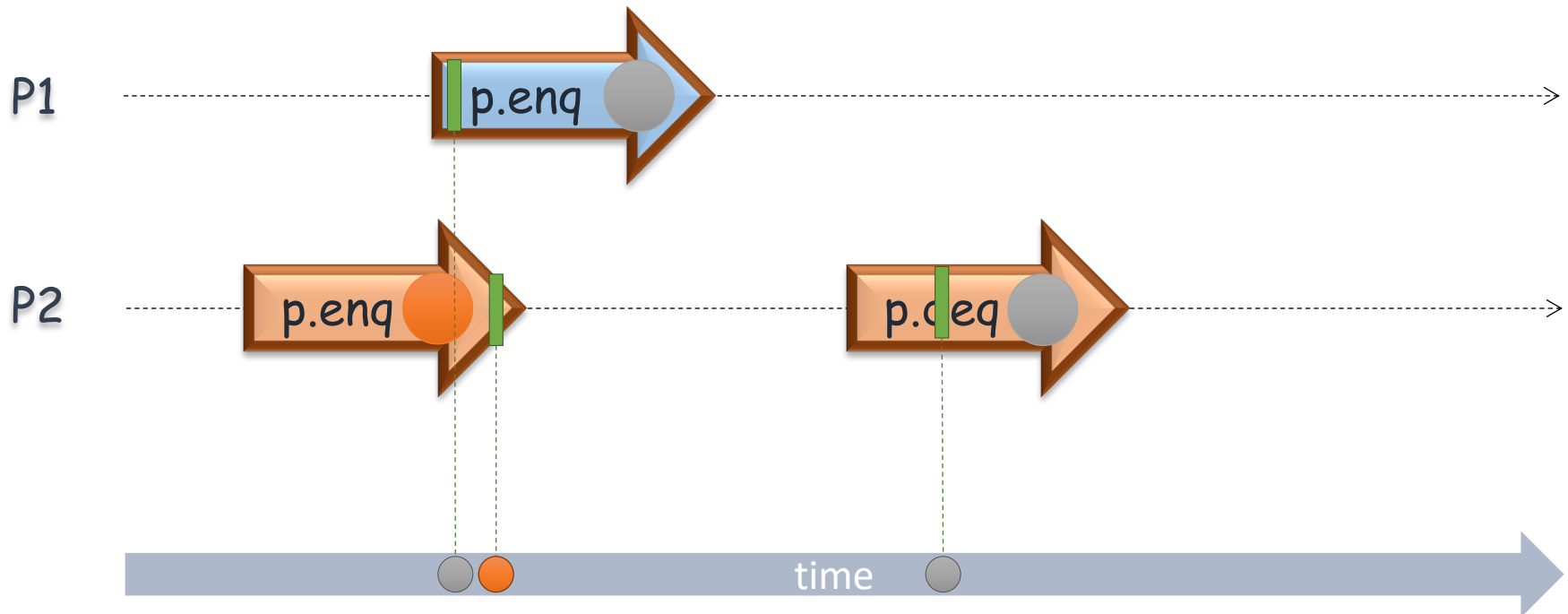
Is this execution sequentially consistent? **Yes**



Sequential Consistency

Is this execution sequentially consistent? **Yes**

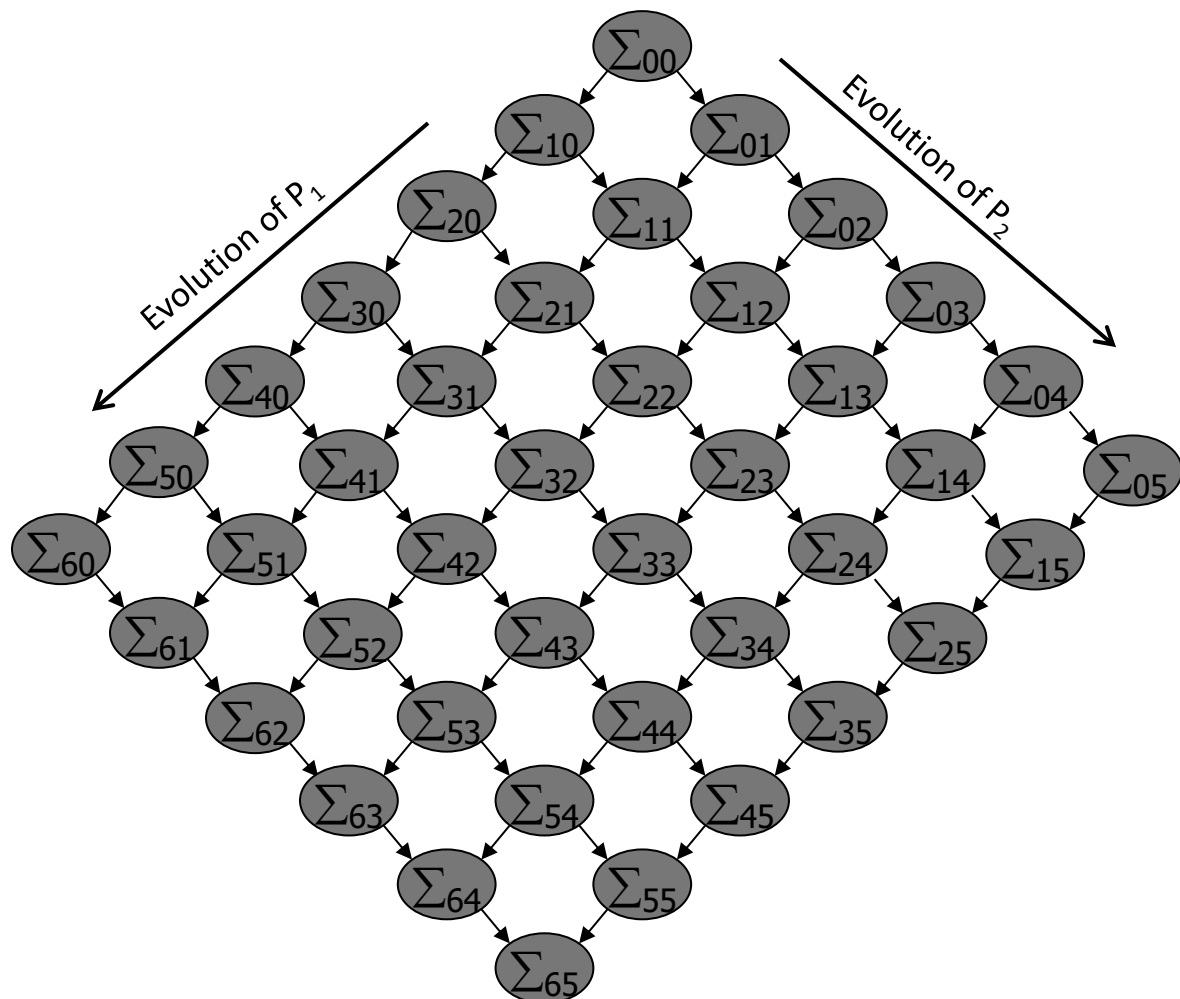
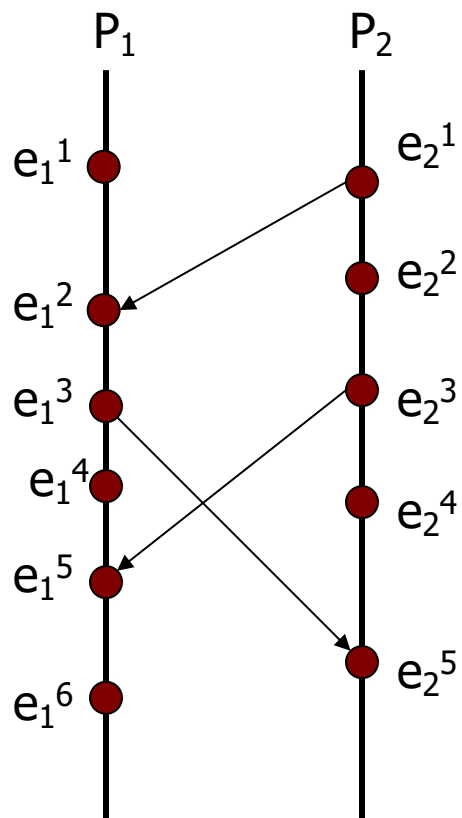
It is also linearizable!



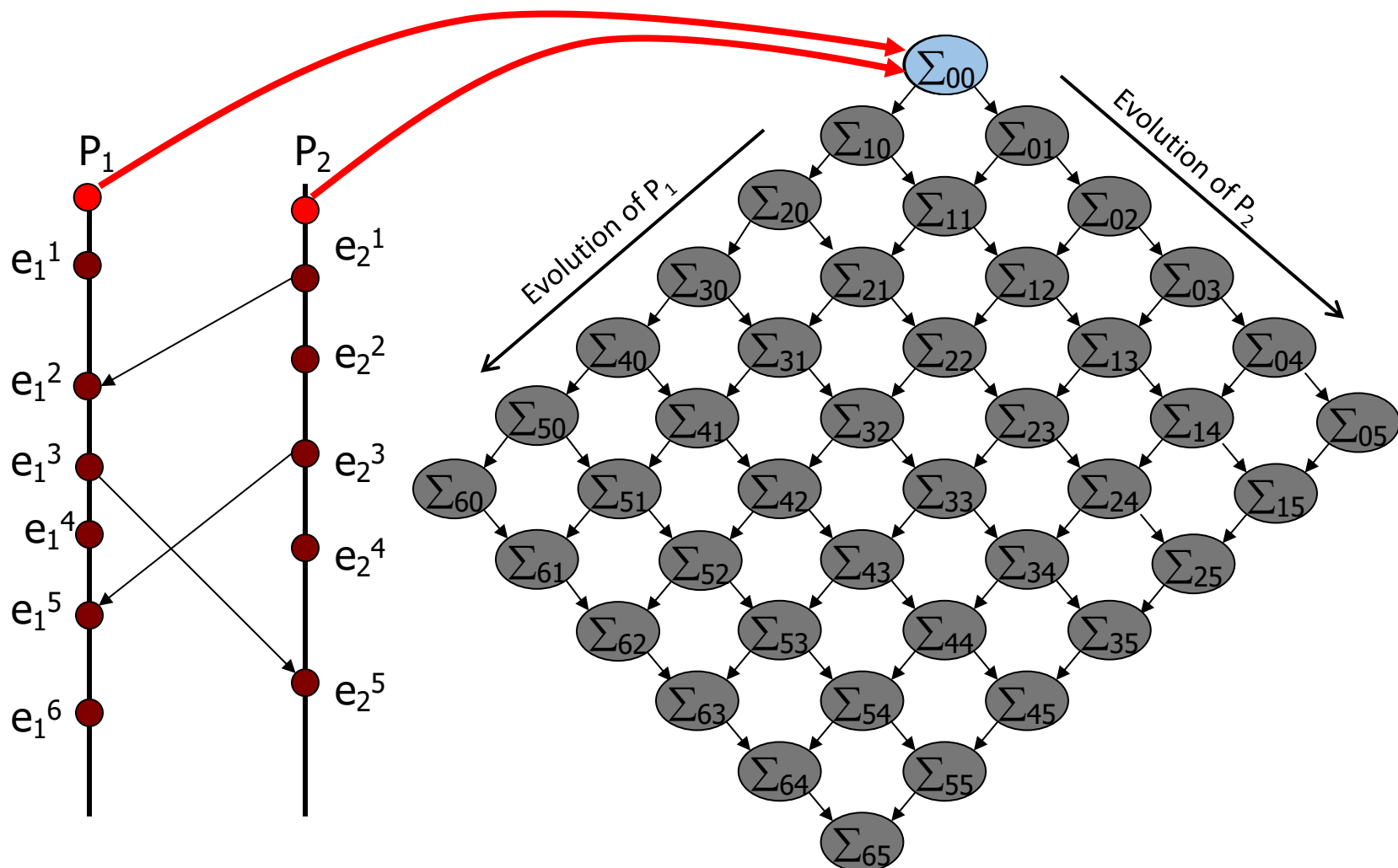
Program State and Runs

- Program state as a N-dimensional lattice
- Runs
 - Consistent
 - Inconsistent

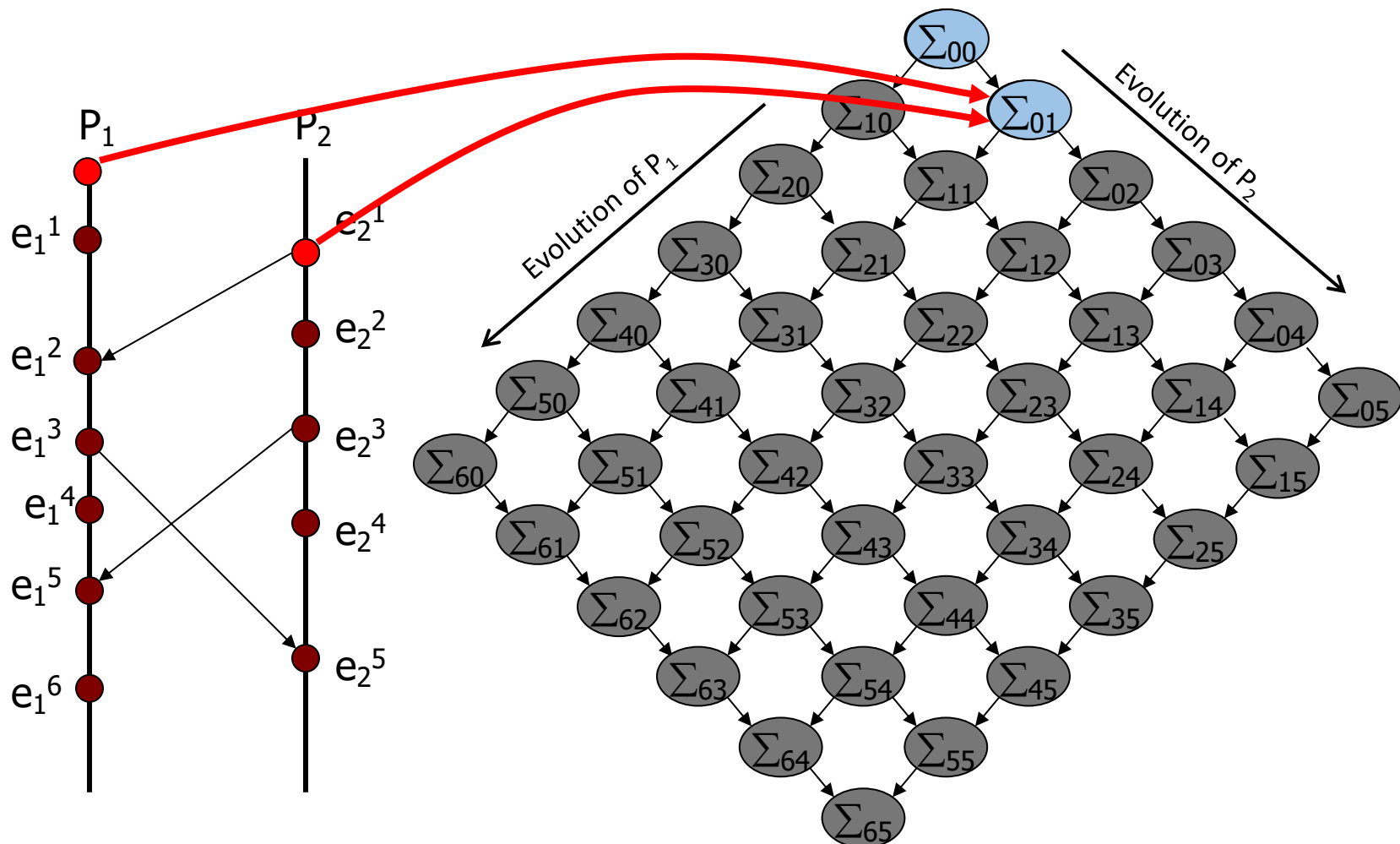
State explosion in concurrent programs



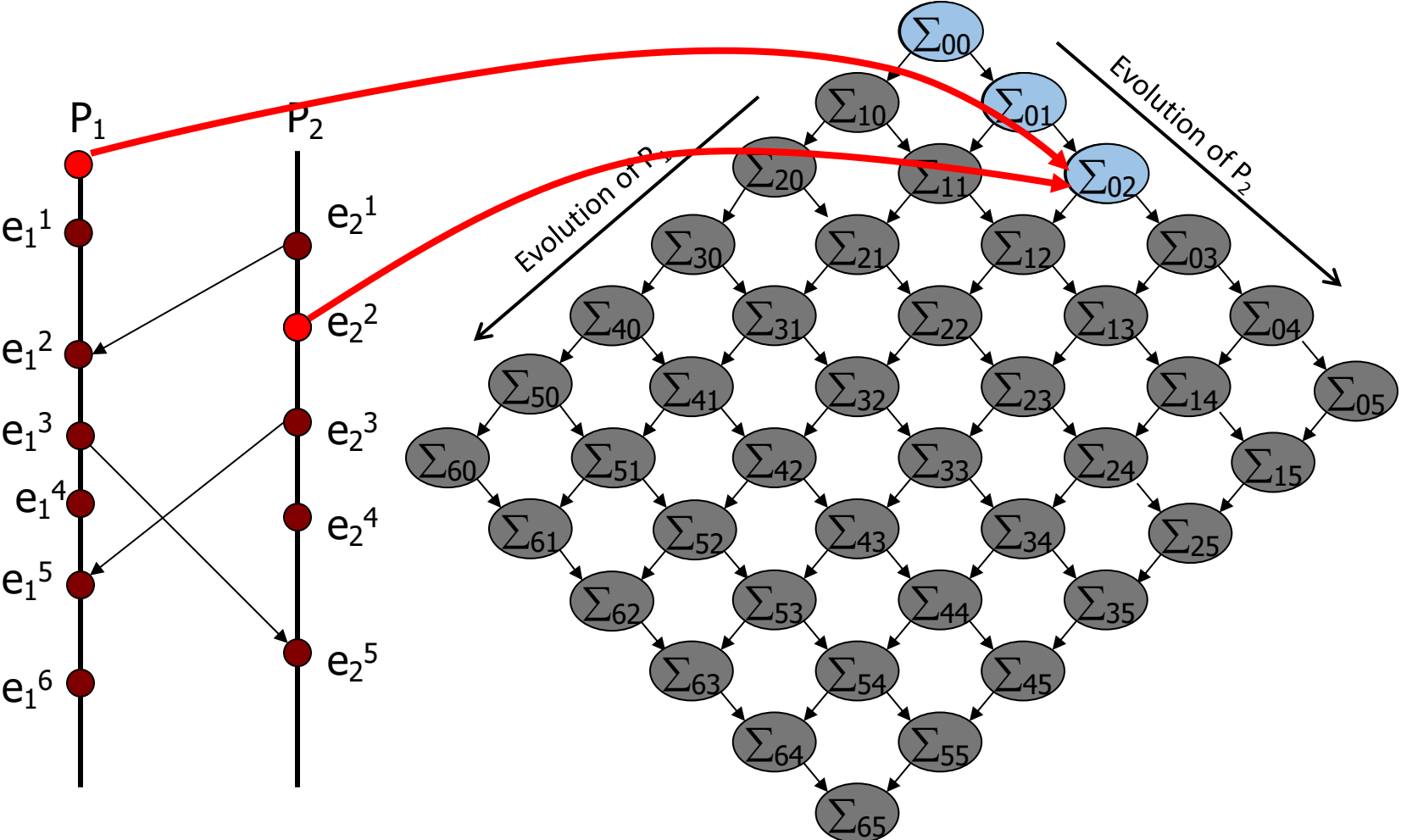
State explosion in concurrent programs



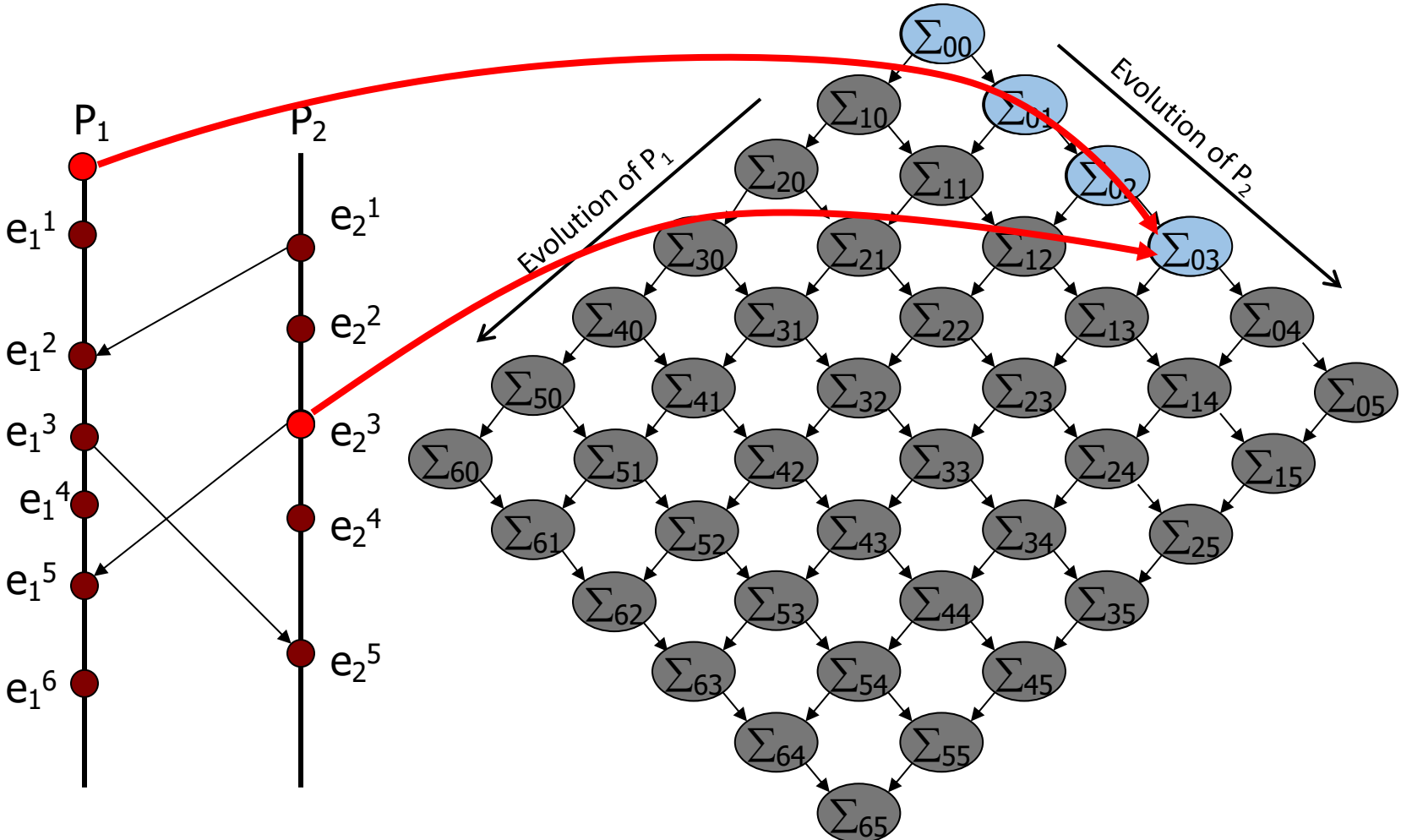
State explosion in concurrent programs



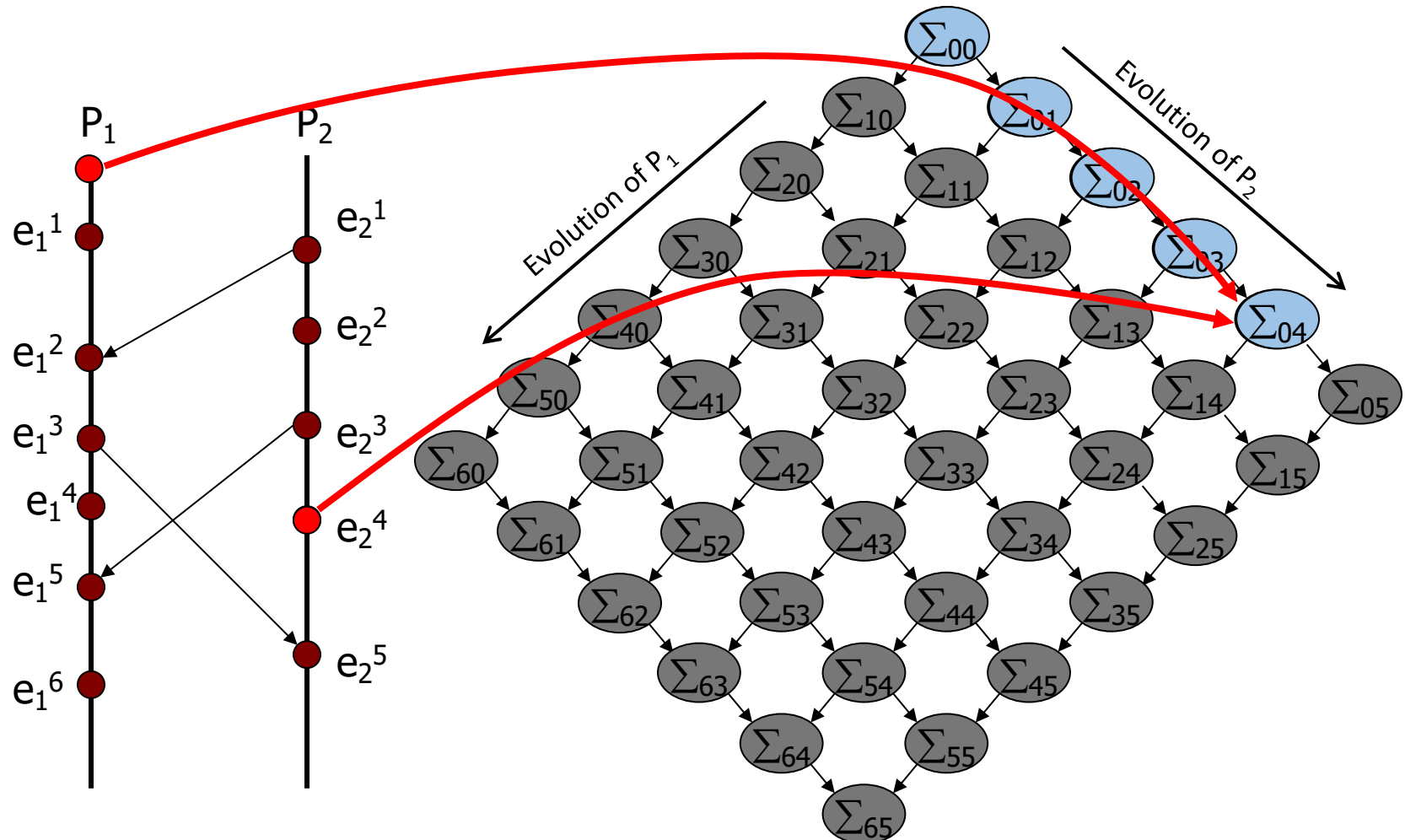
State explosion in concurrent programs



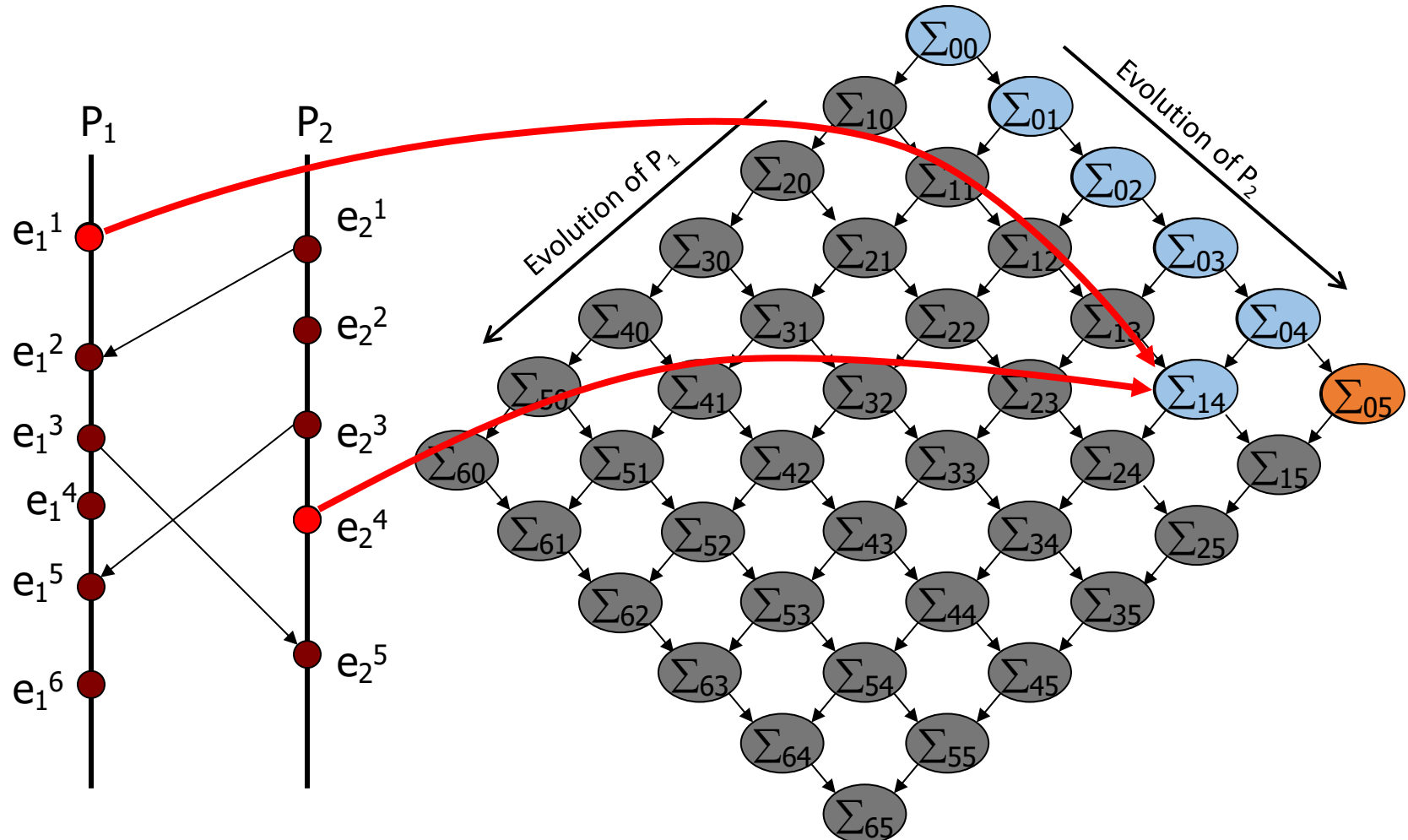
State explosion in concurrent programs



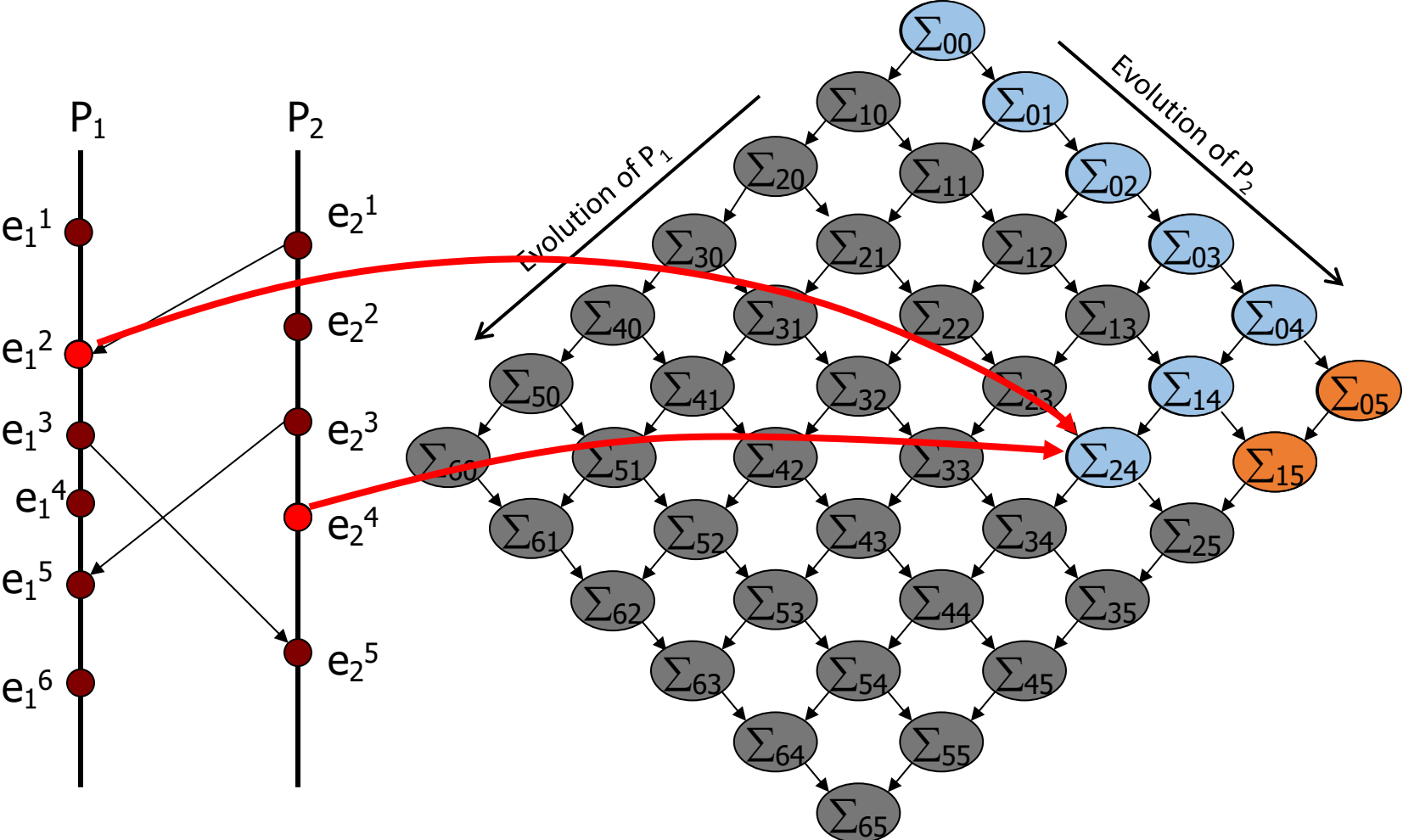
State explosion in concurrent programs



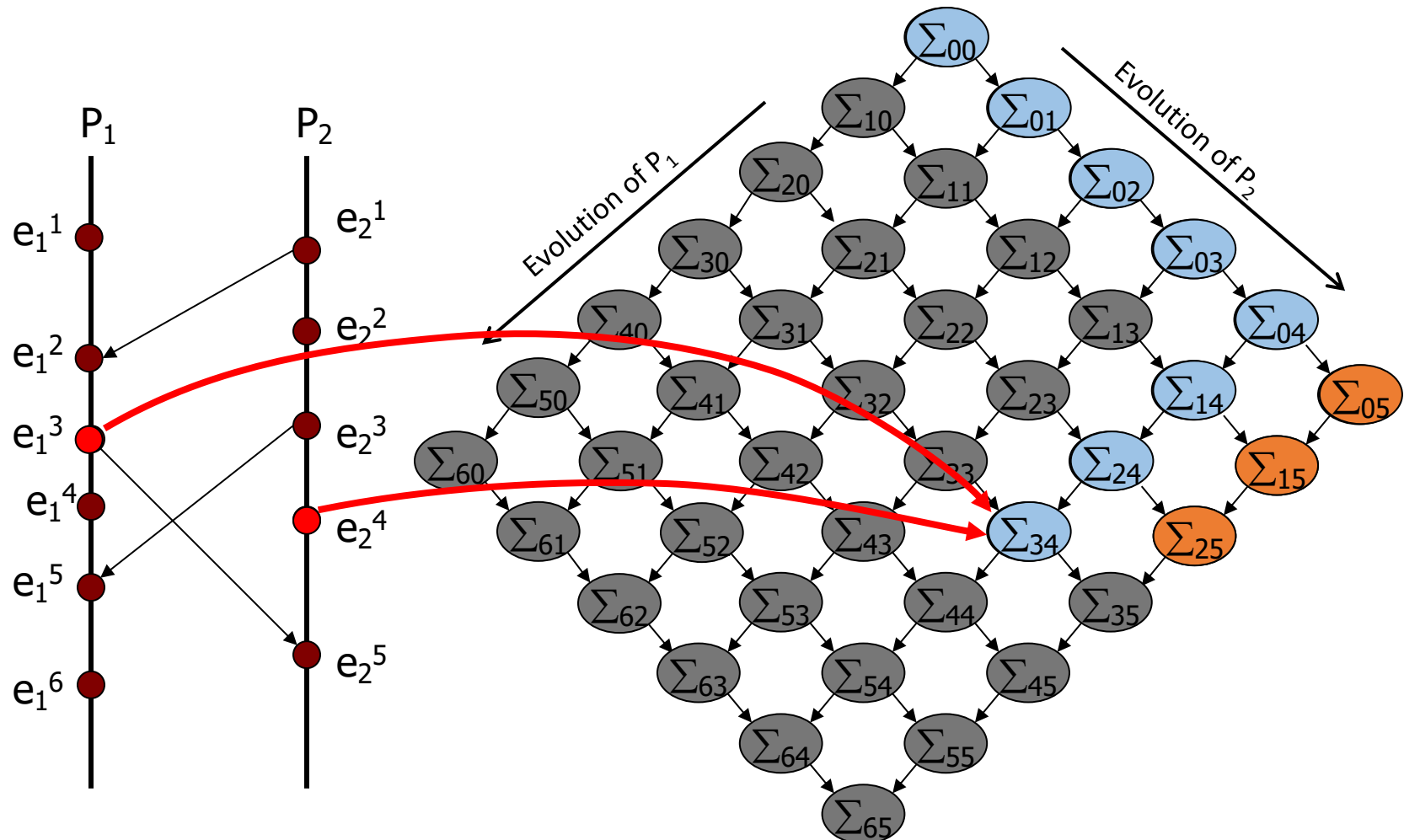
State explosion in concurrent programs



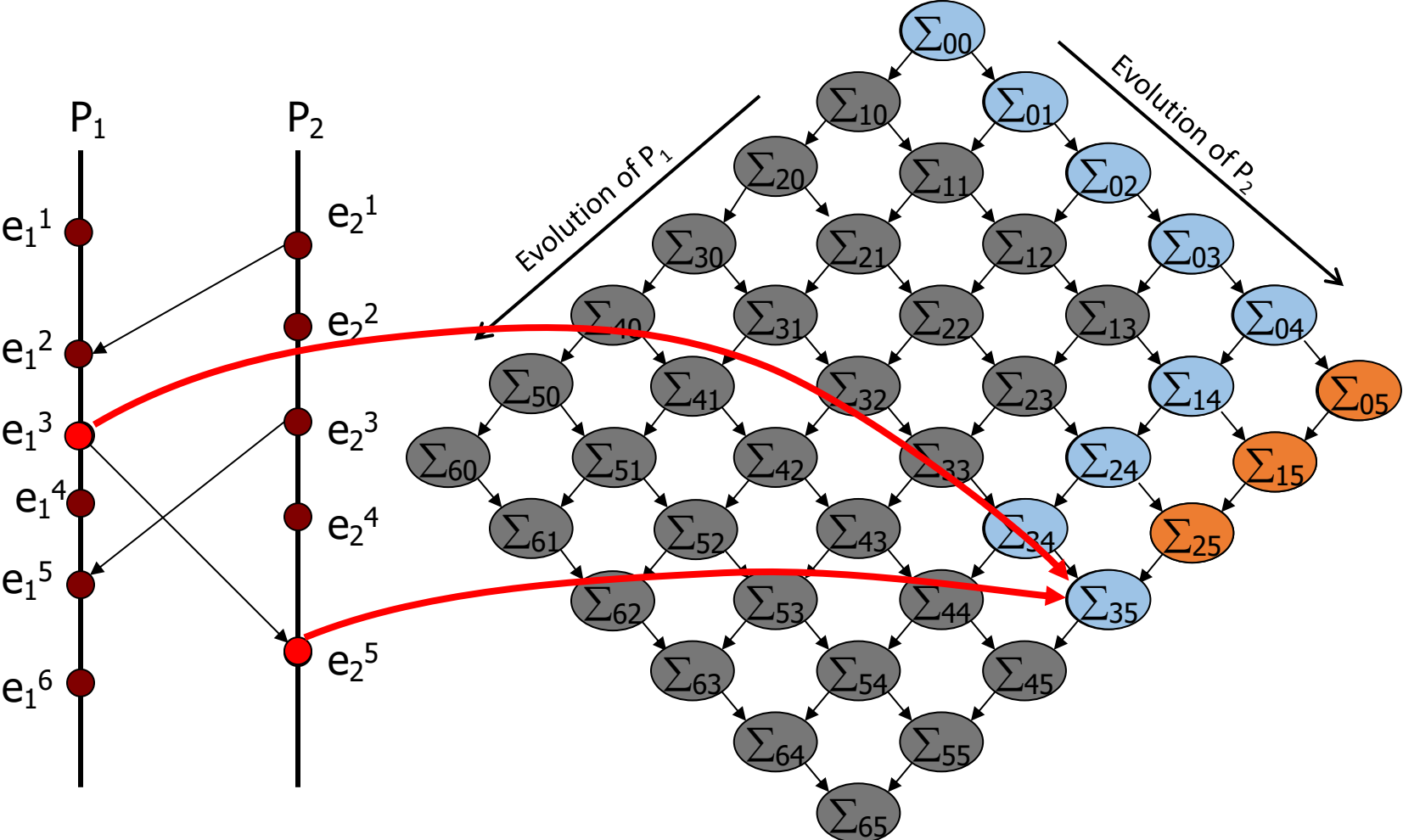
State explosion in concurrent programs



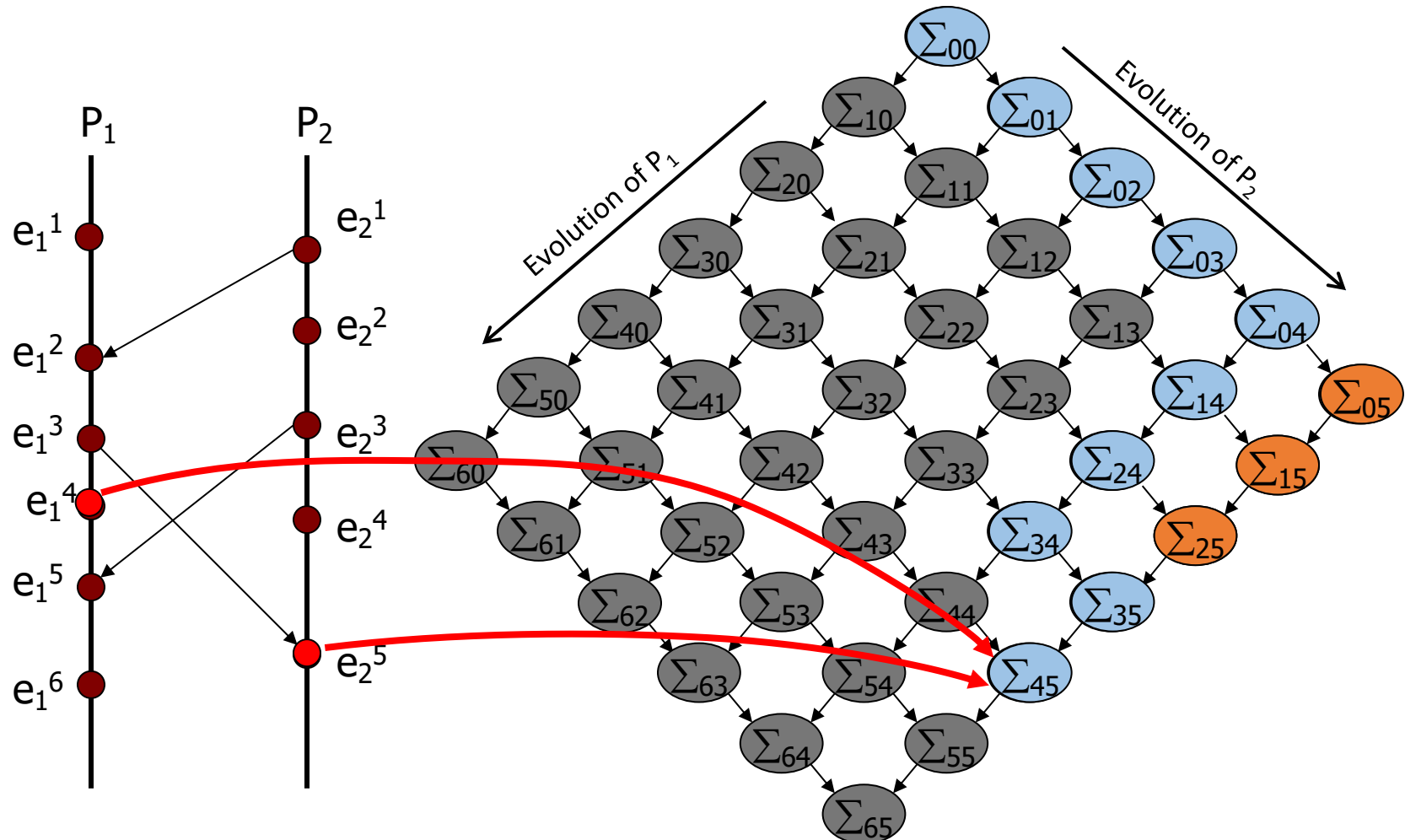
State explosion in concurrent programs



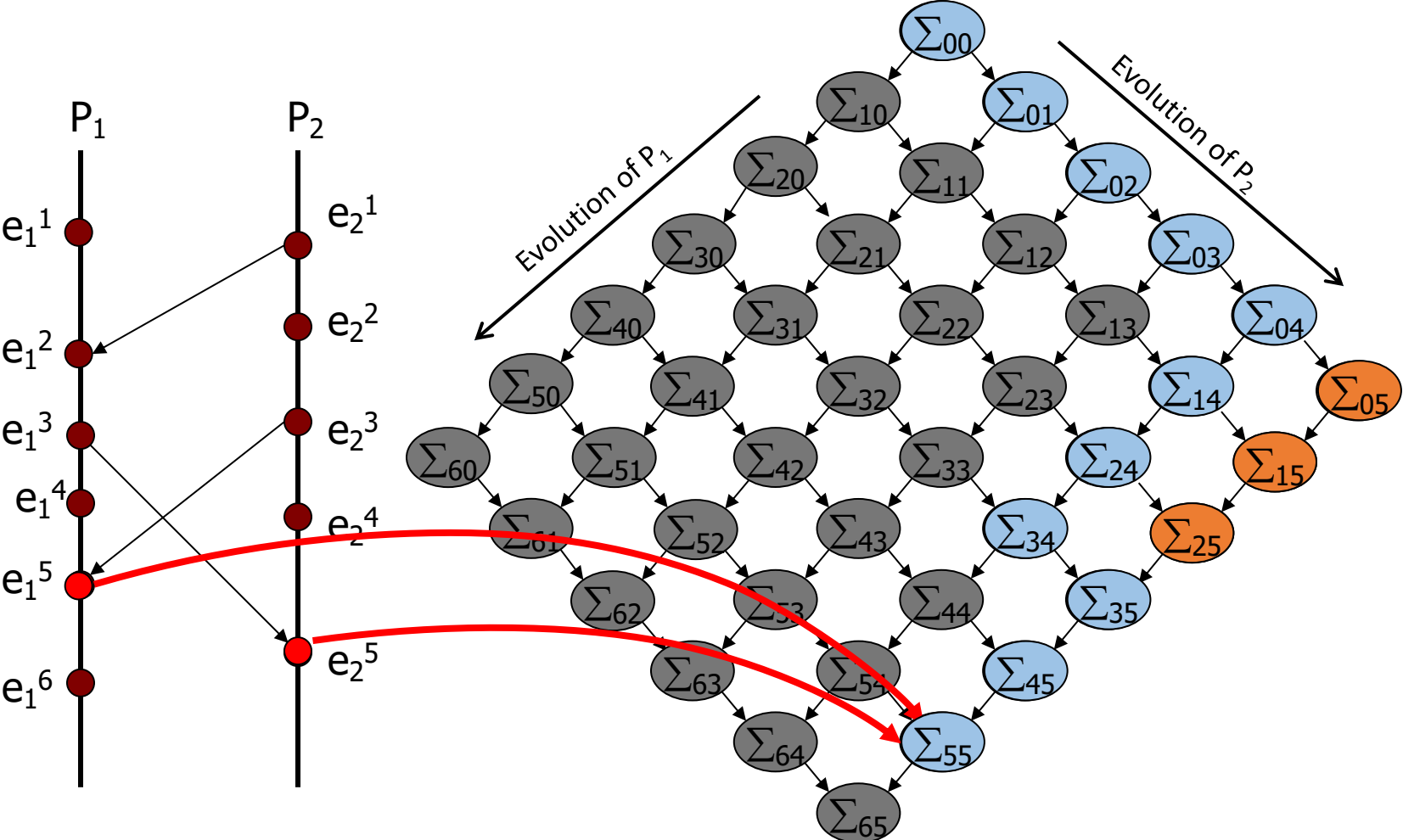
State explosion in concurrent programs



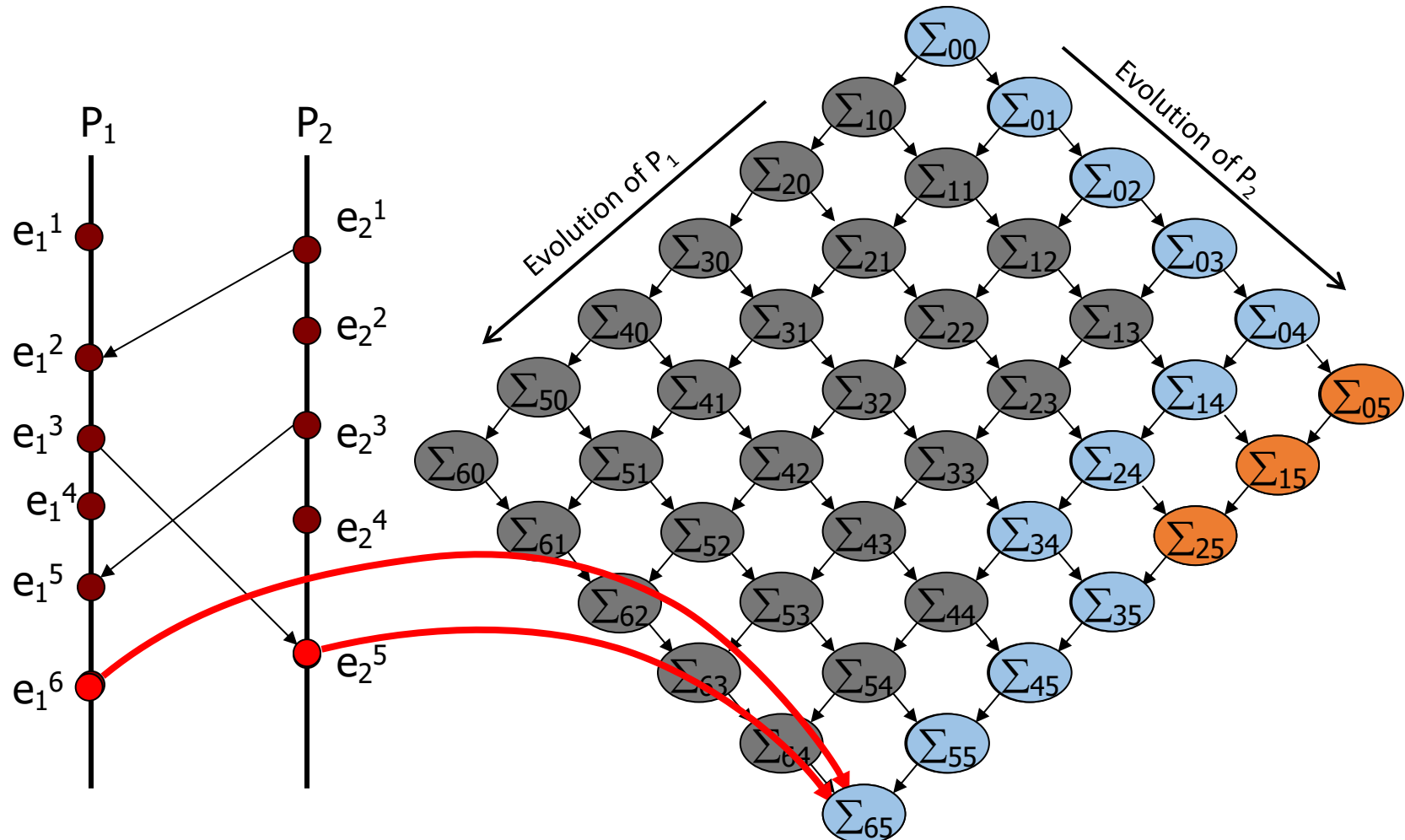
State explosion in concurrent programs



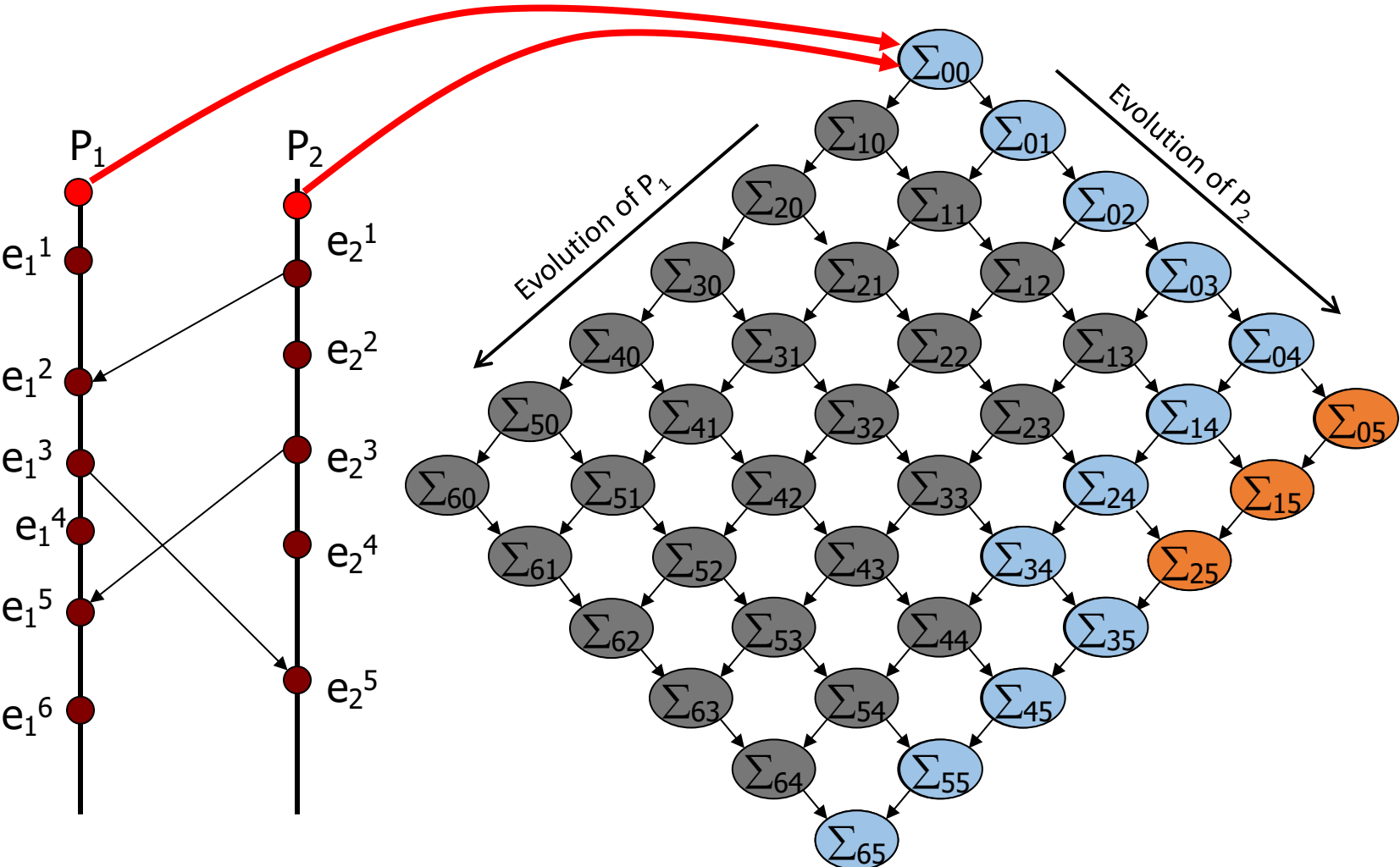
State explosion in concurrent programs



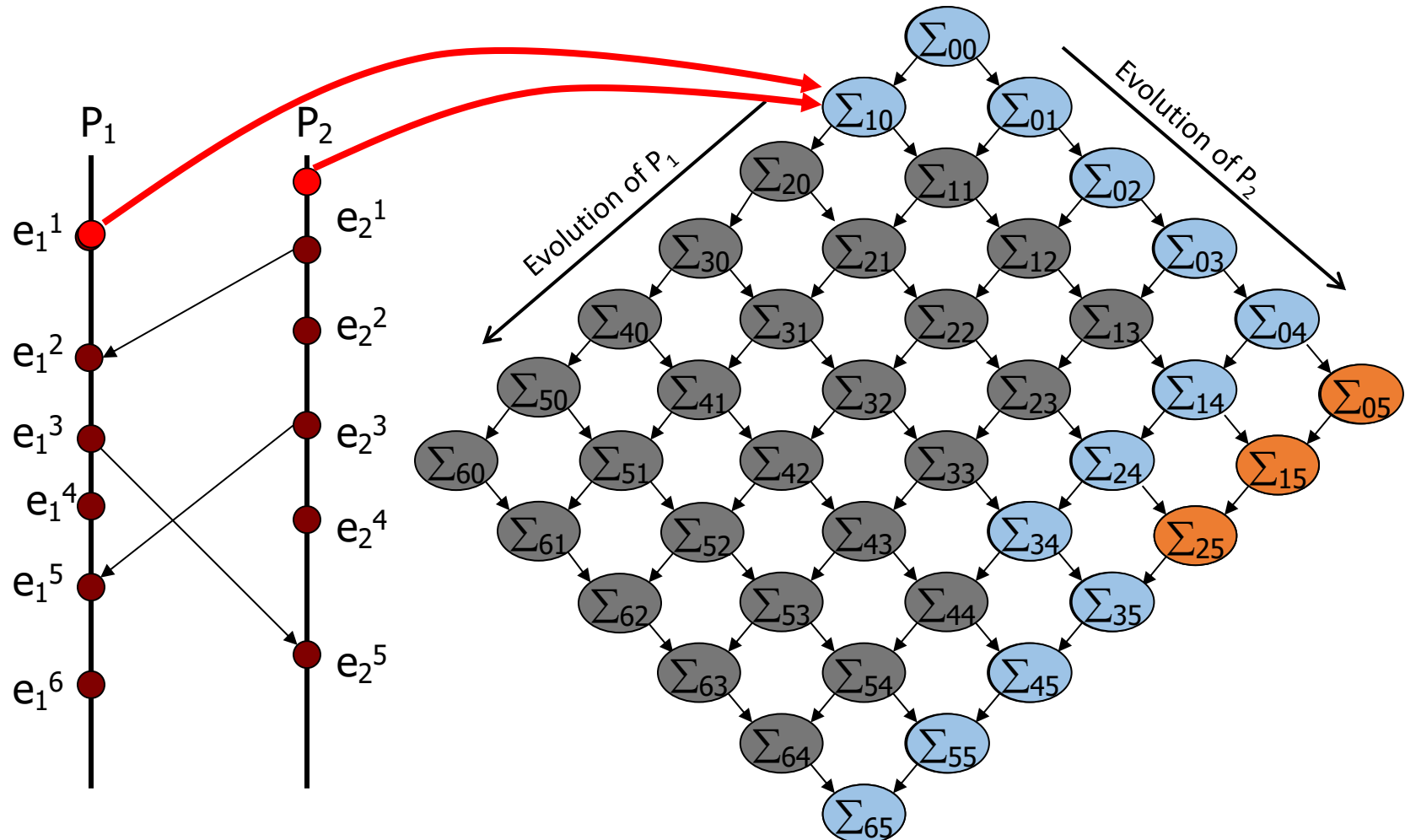
State explosion in concurrent programs



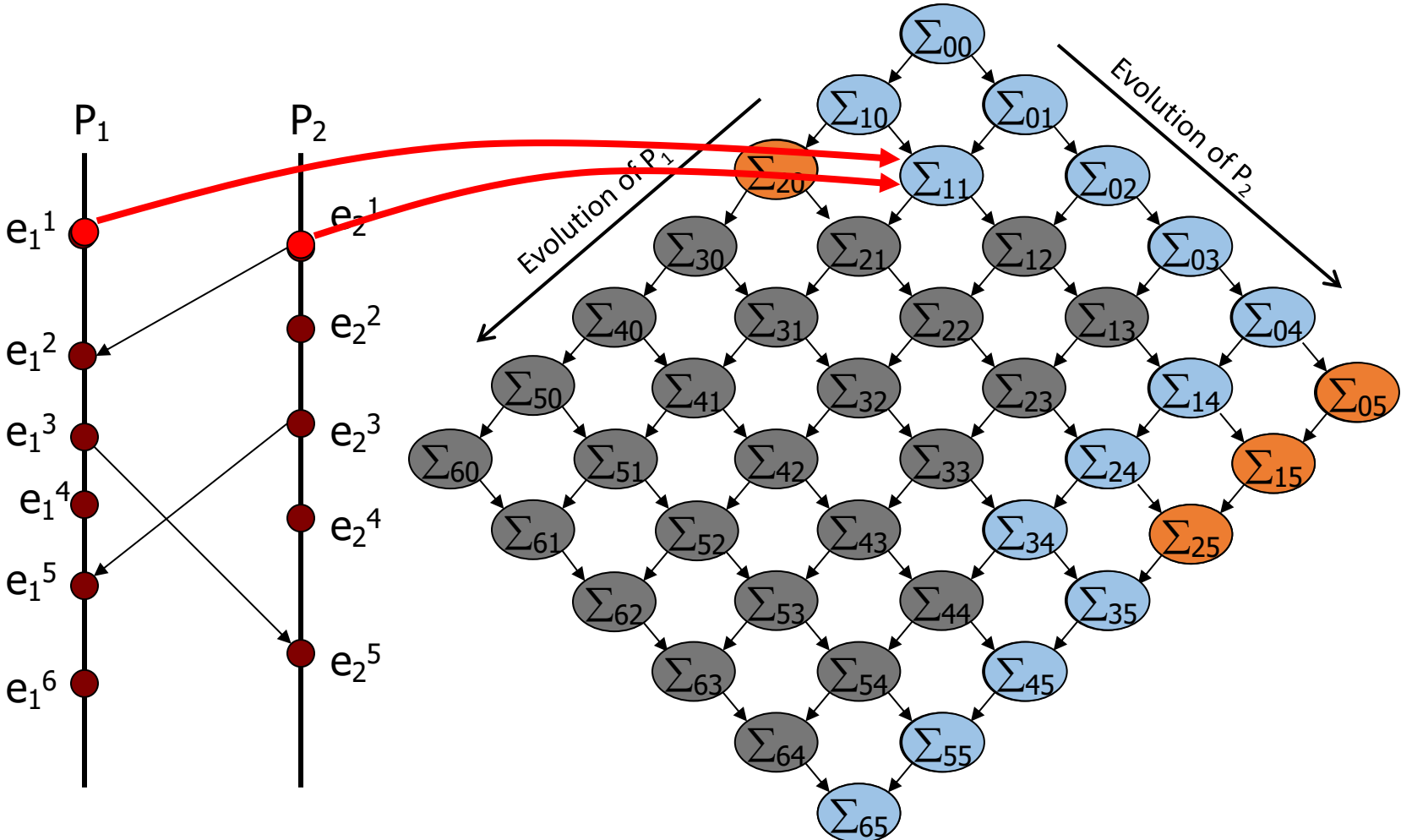
State explosion in concurrent programs



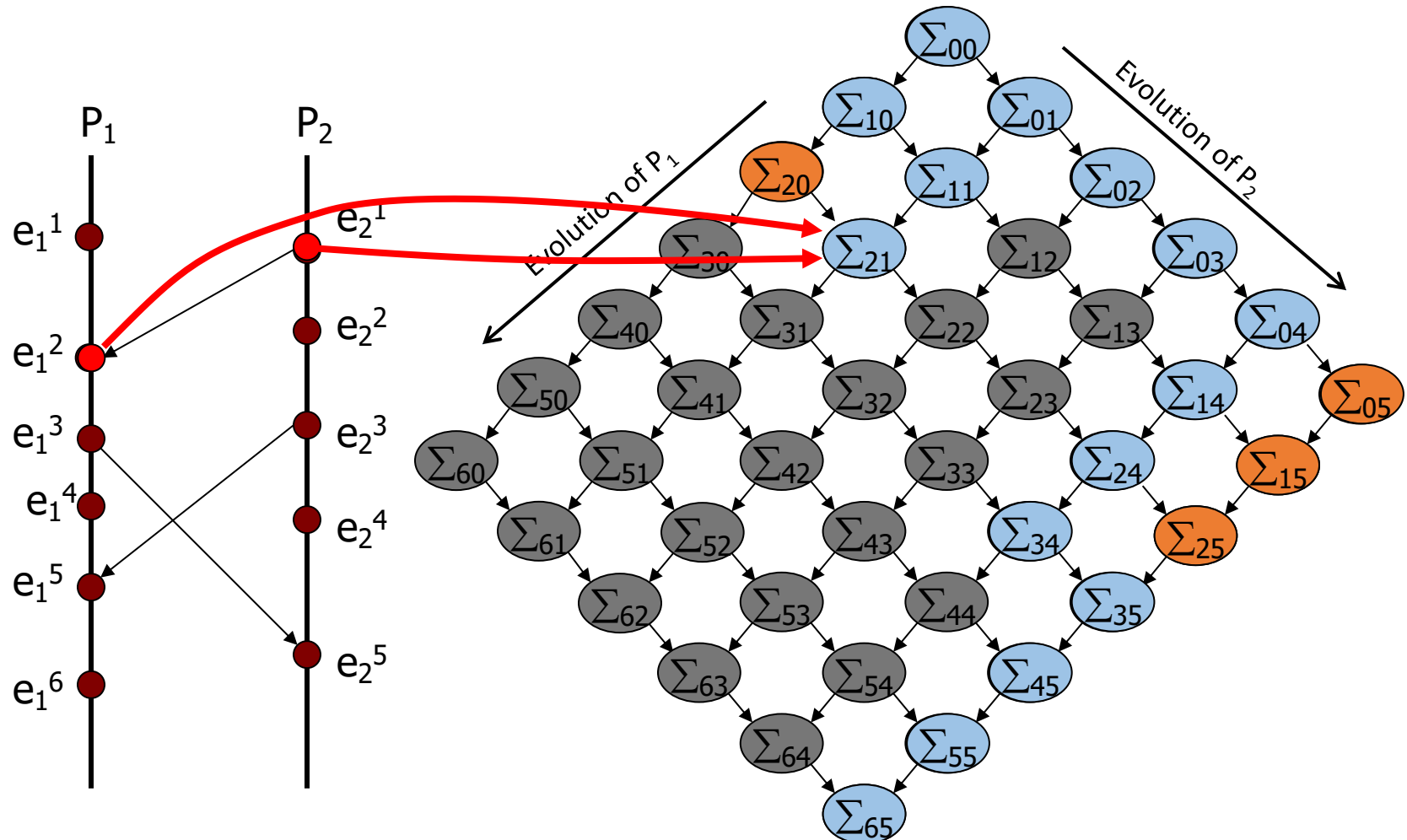
State explosion in concurrent programs



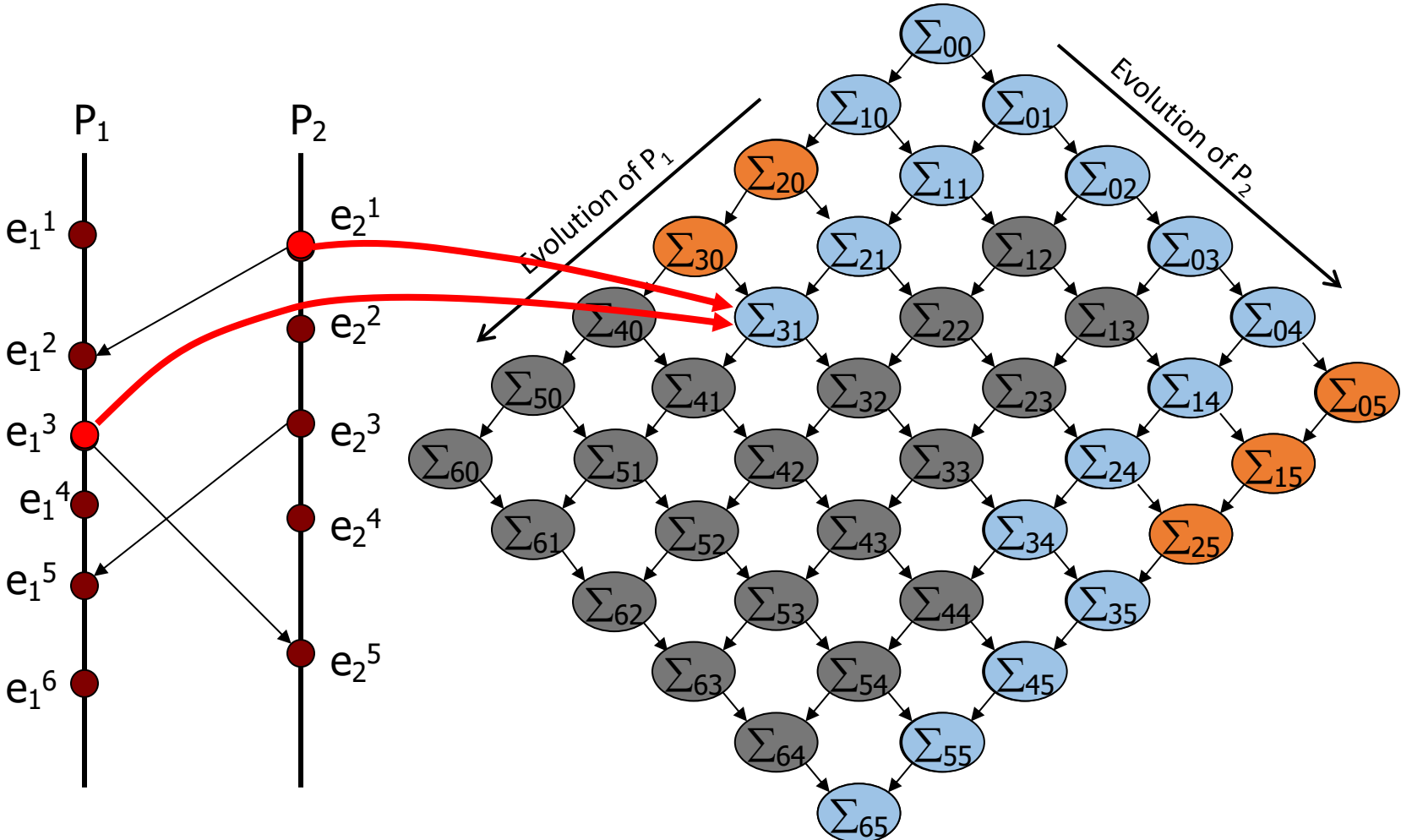
State explosion in concurrent programs



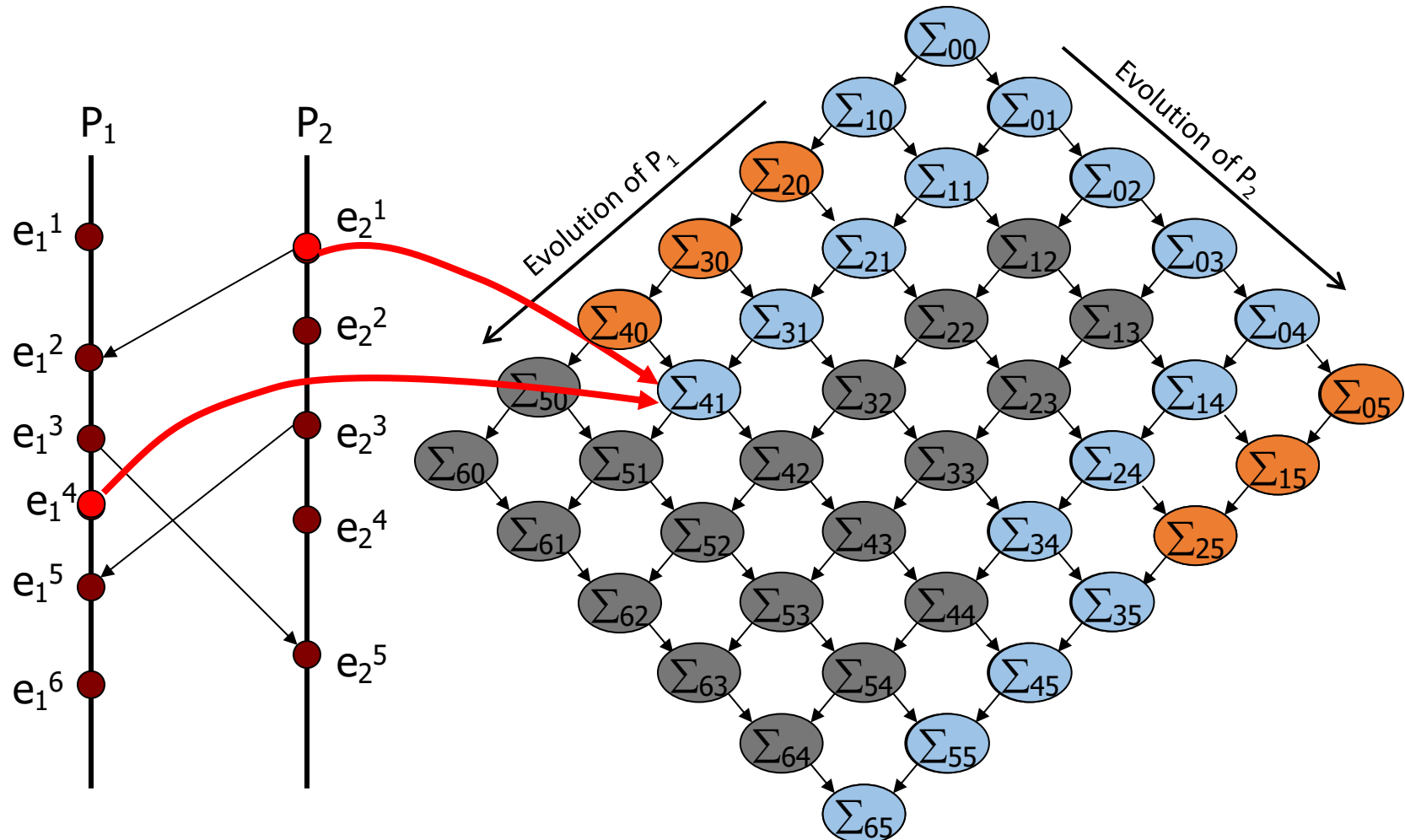
State explosion in concurrent programs



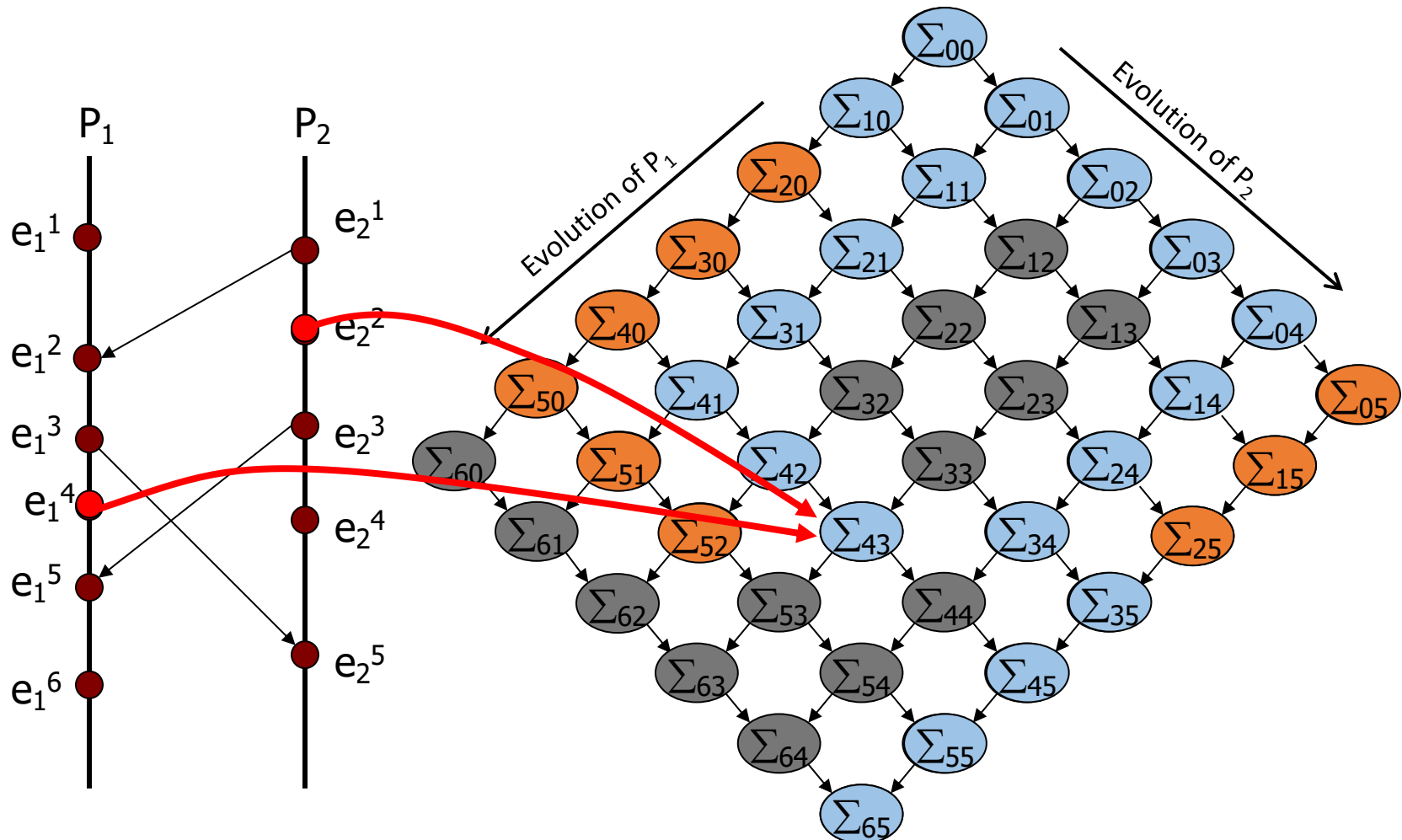
State explosion in concurrent programs



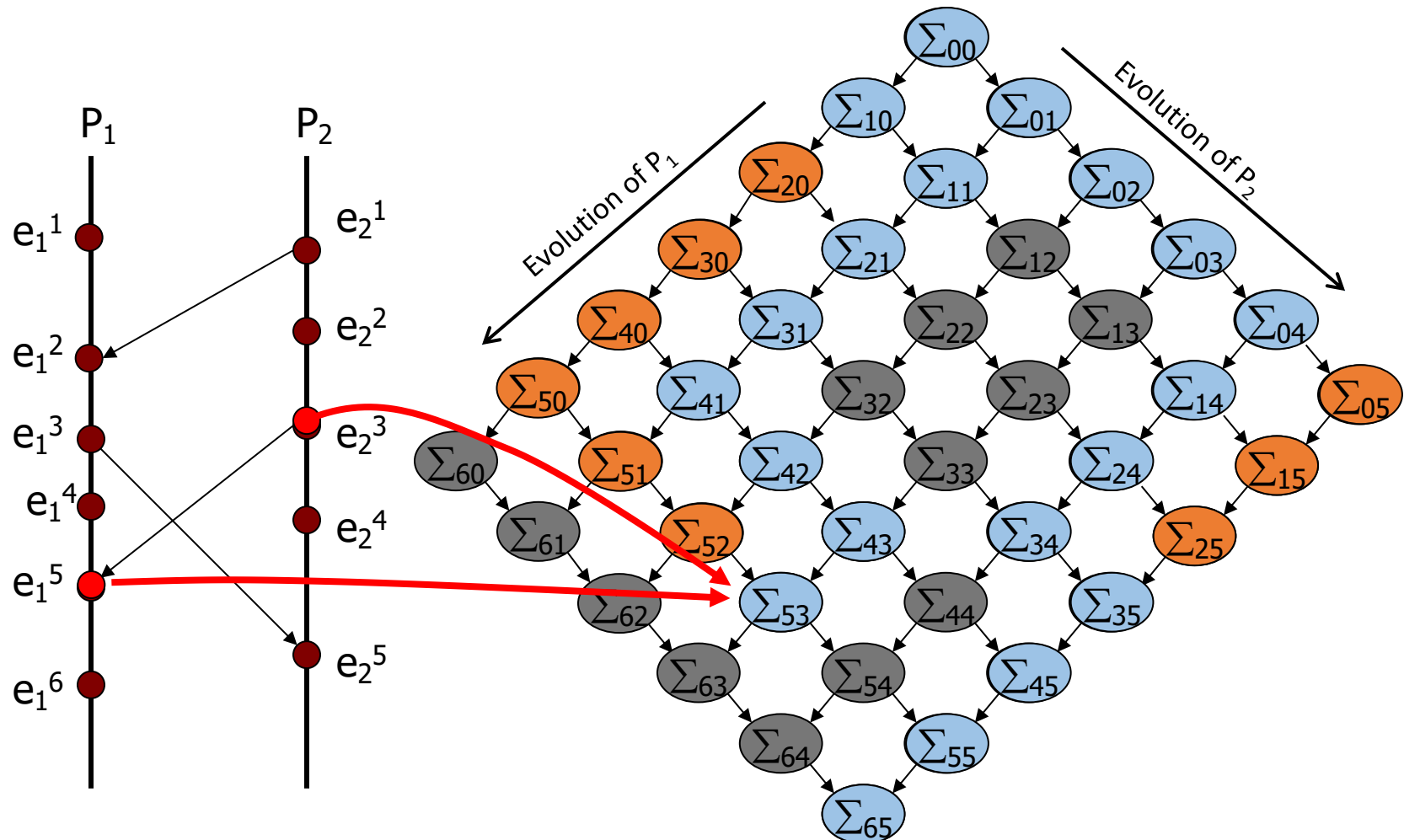
State explosion in concurrent programs



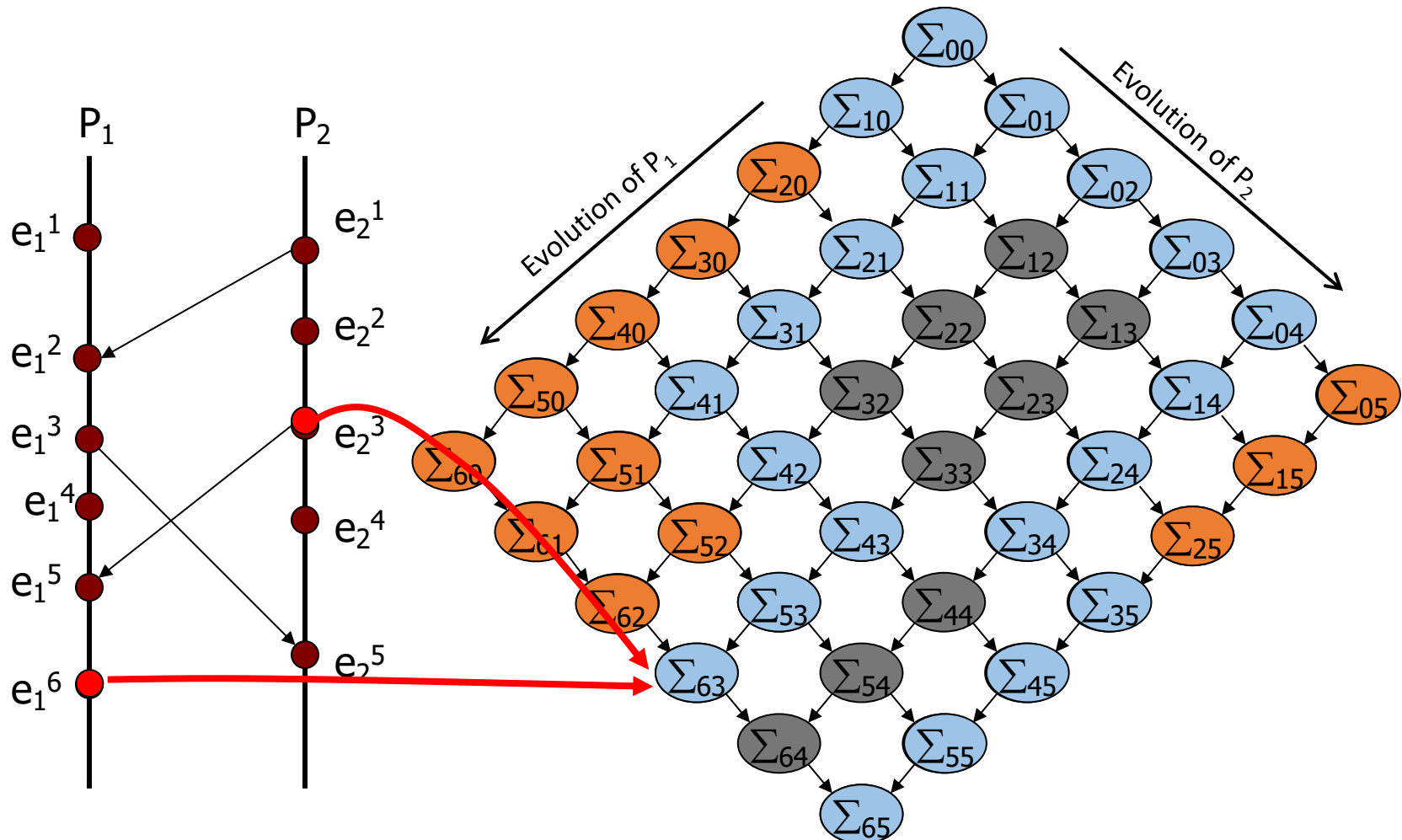
State explosion in concurrent programs



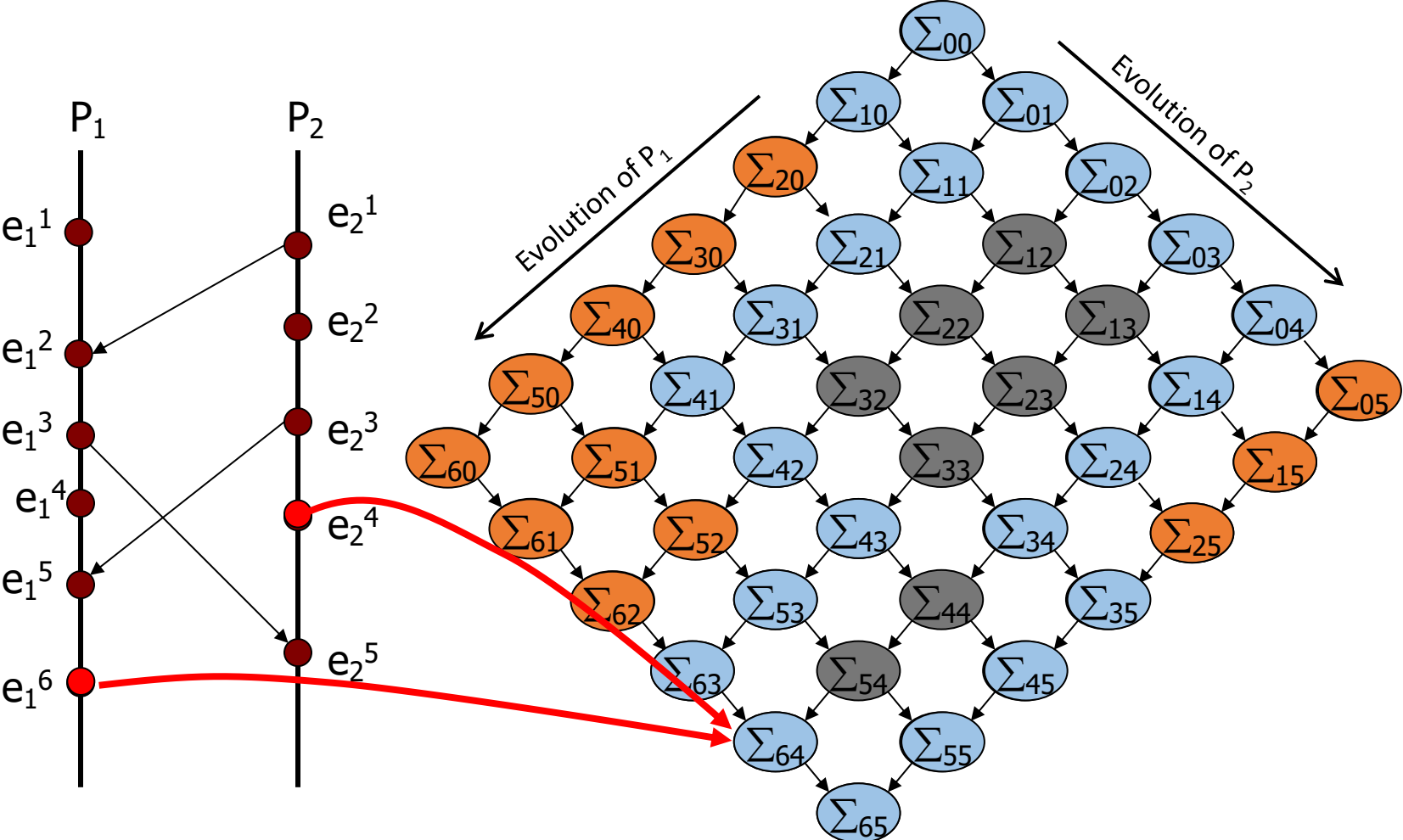
State explosion in concurrent programs



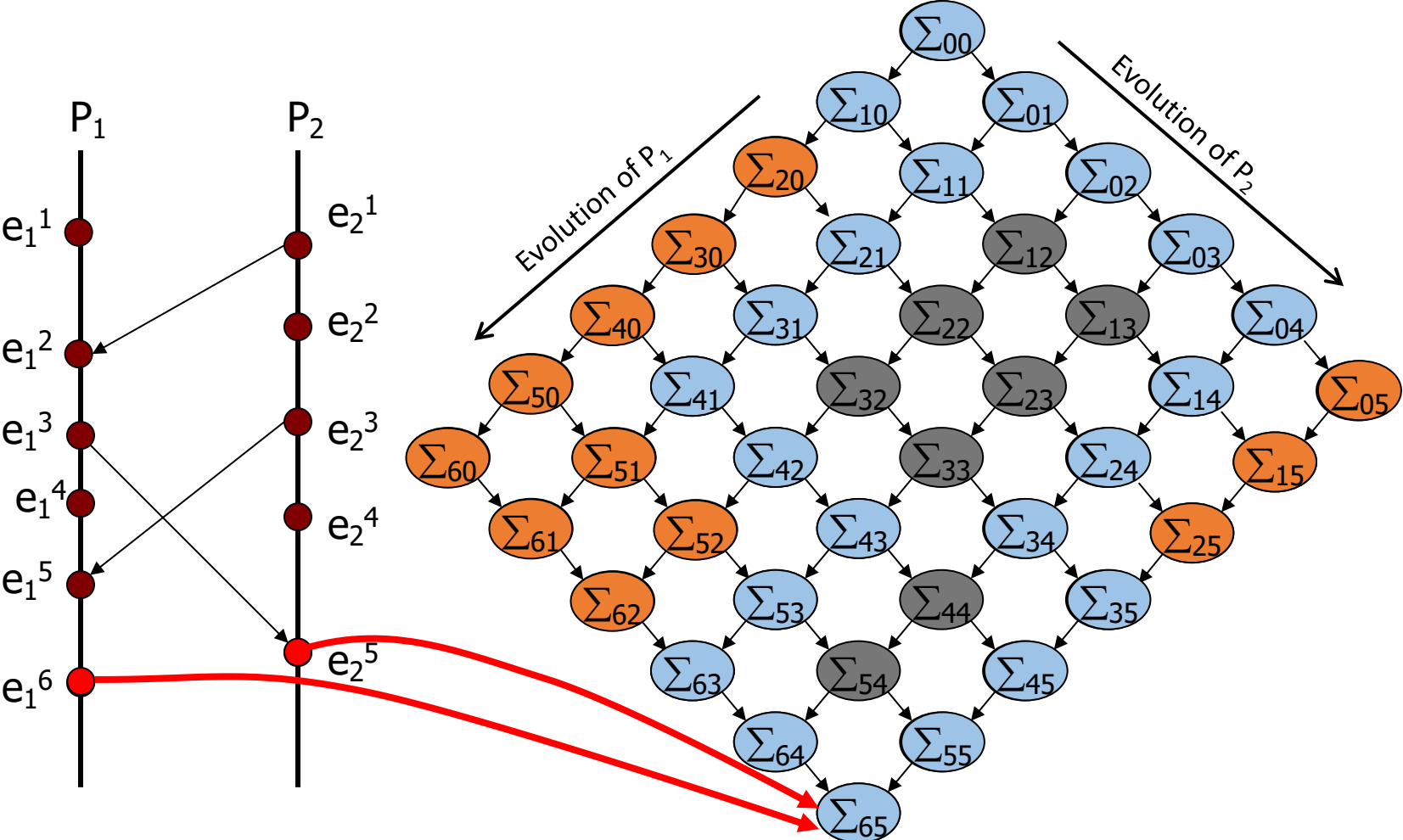
State explosion in concurrent programs



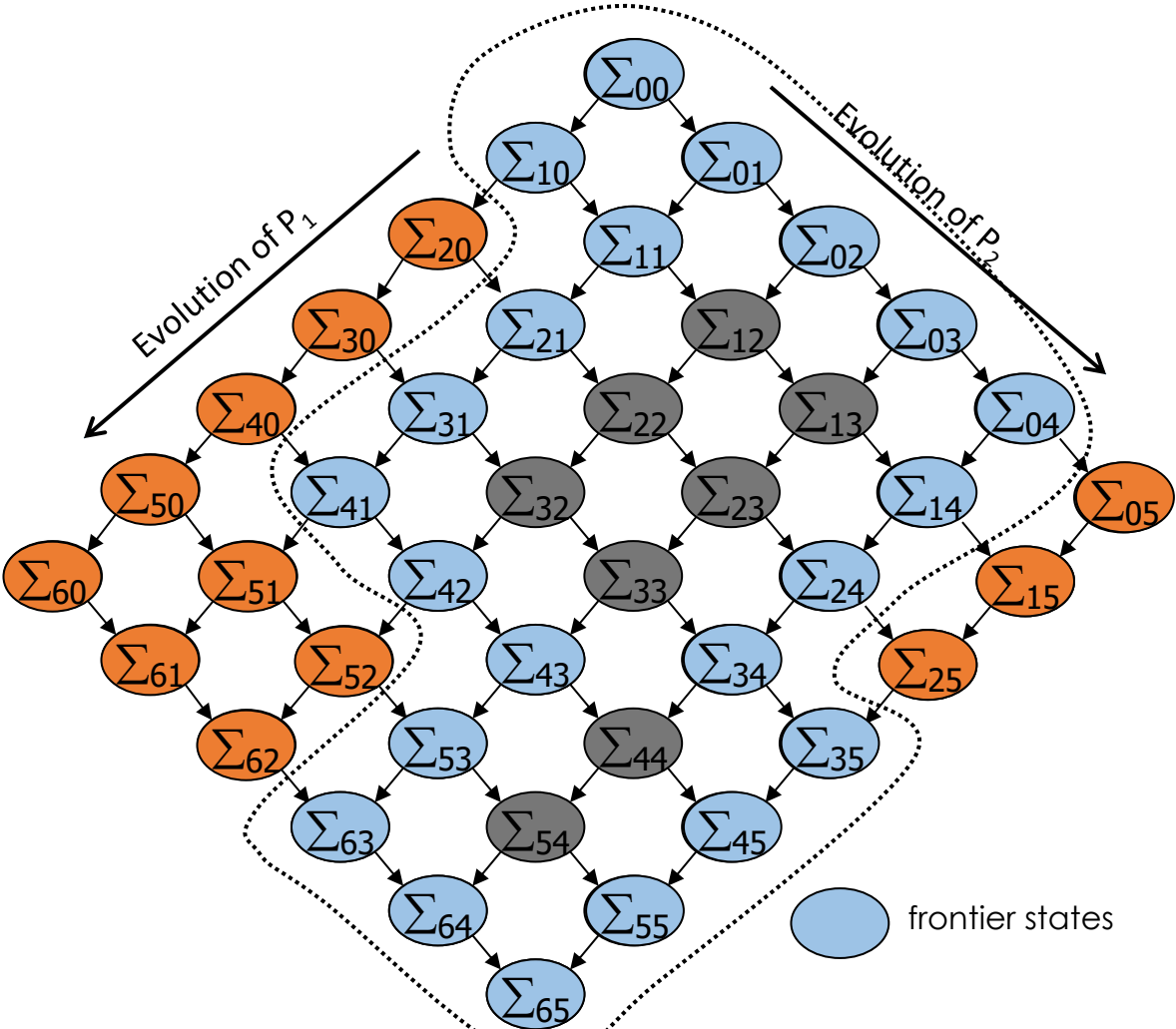
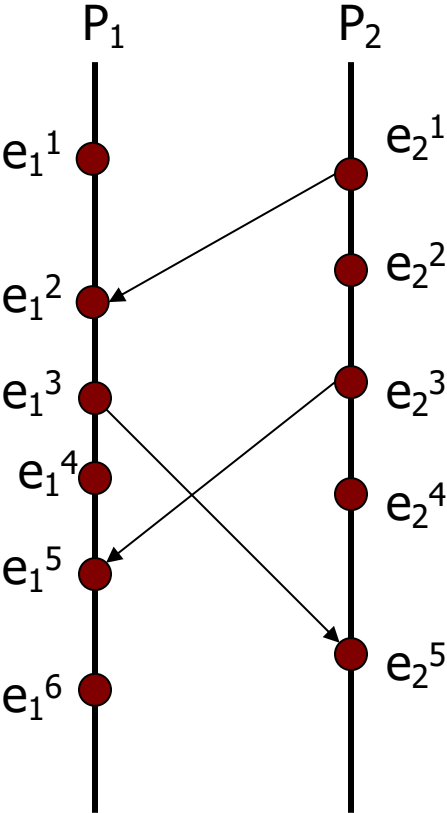
State explosion in concurrent programs



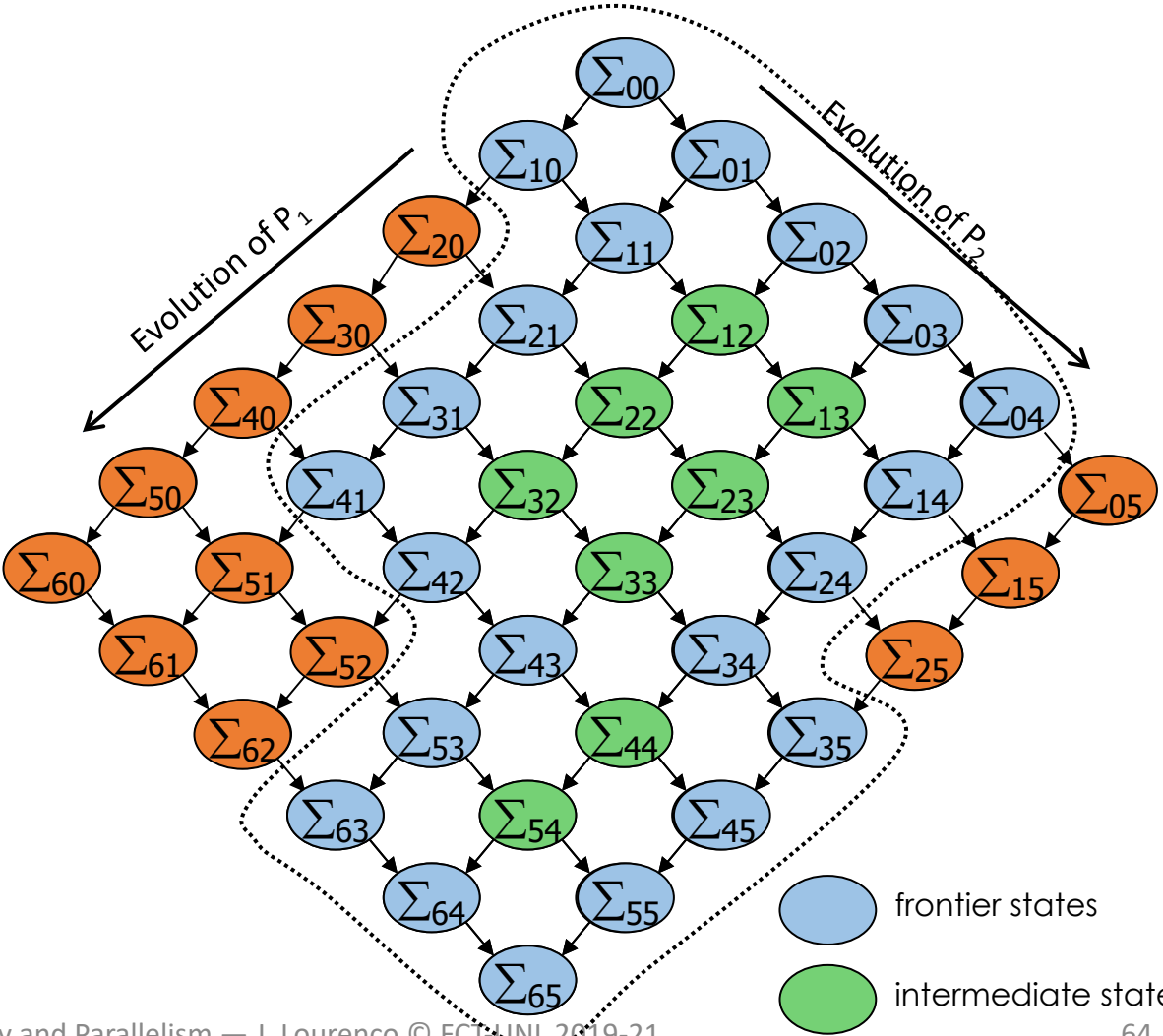
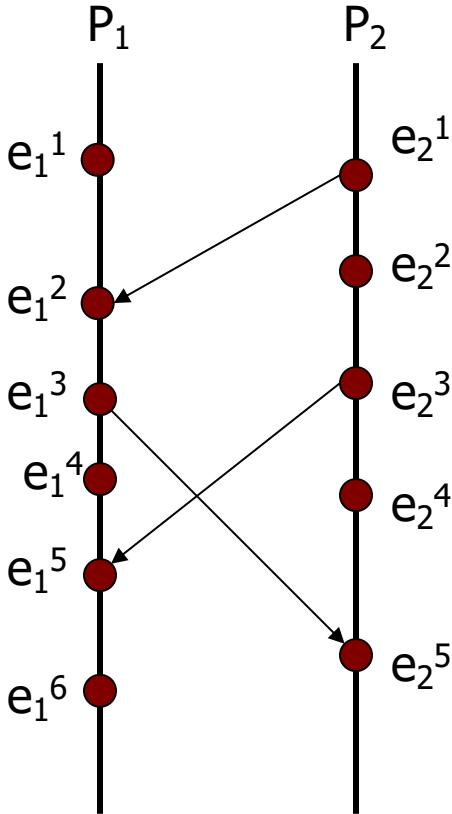
State explosion in concurrent programs



State explosion in concurrent programs



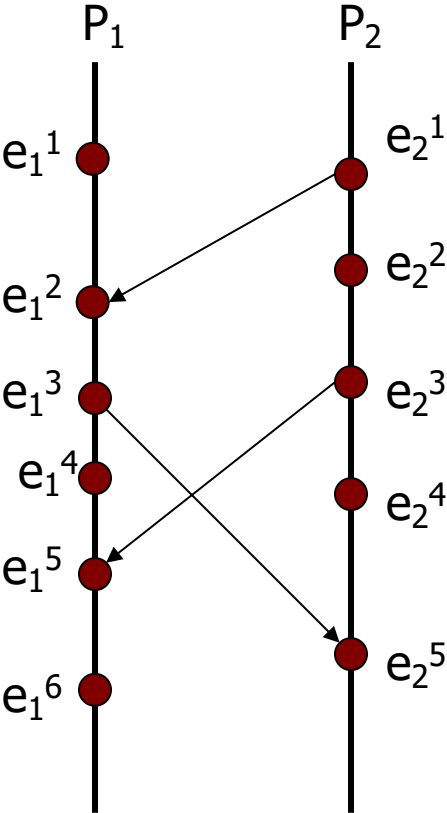
State explosion in concurrent programs



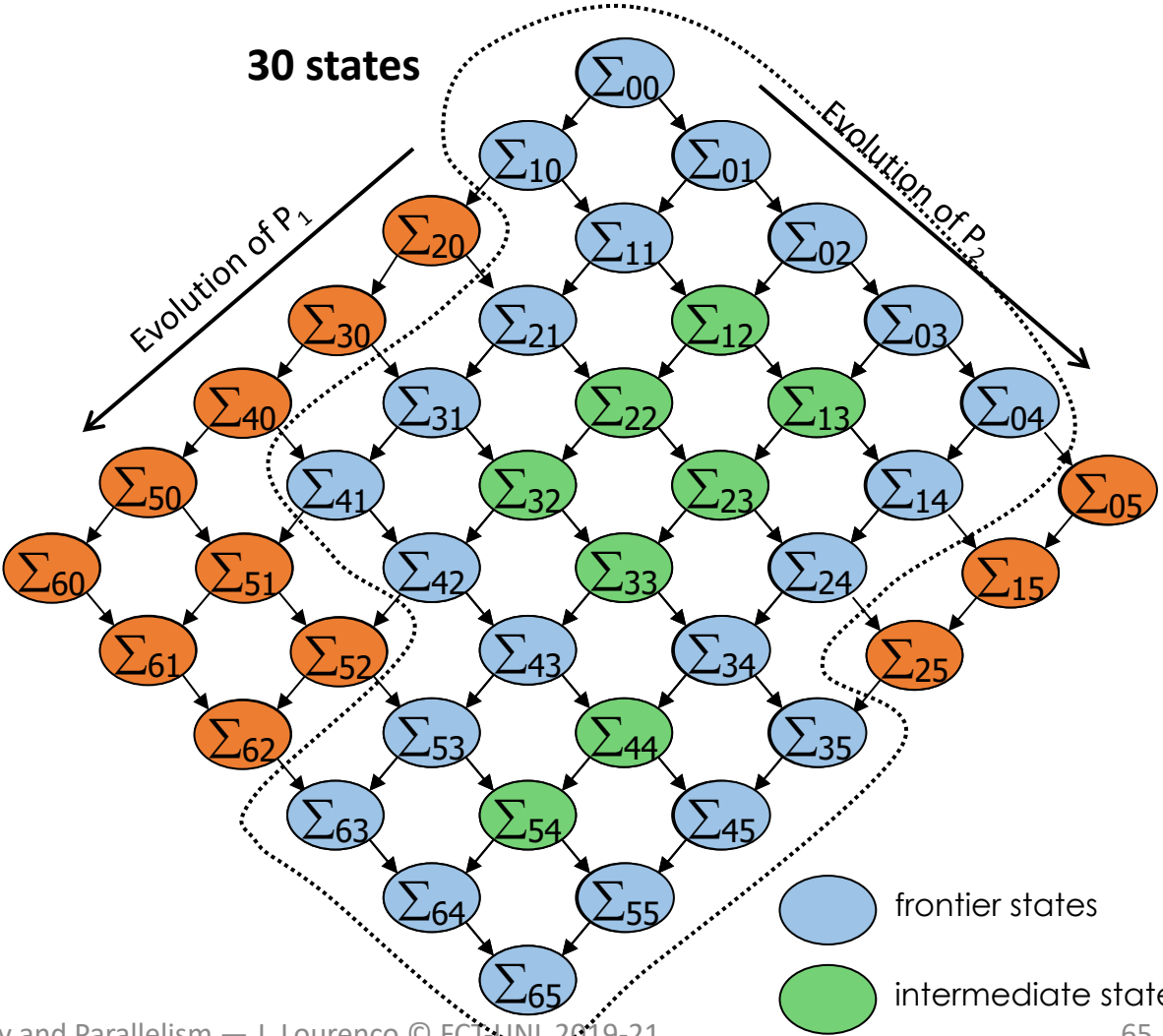
State explosion in concurrent programs

7 states

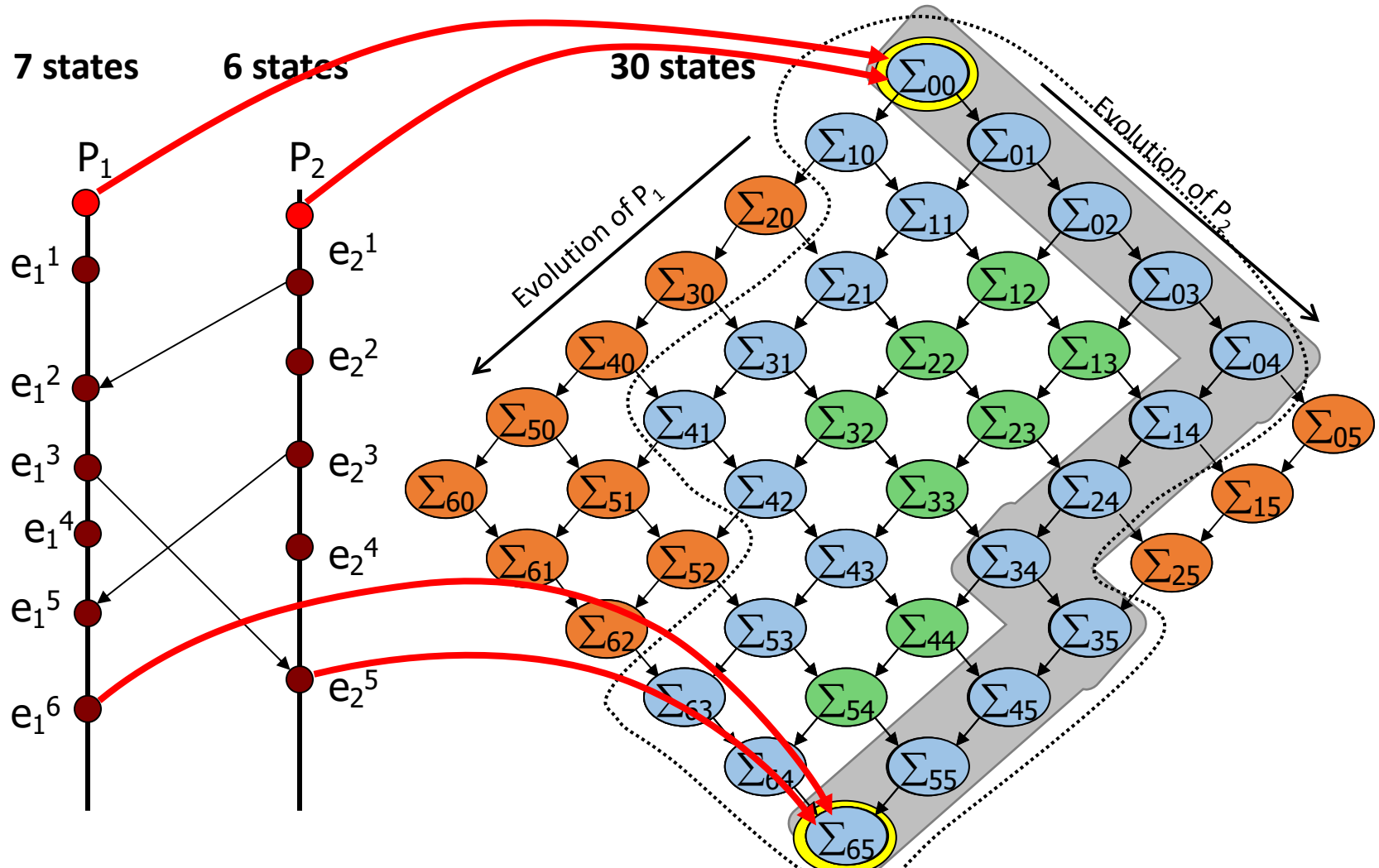
6 states



30 states

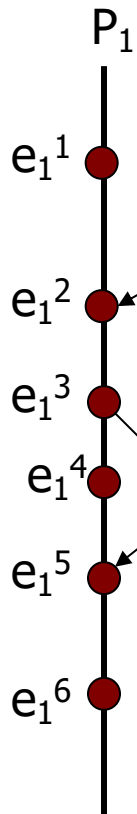


Consistent run: valid path Σ_{00} to Σ_{65}

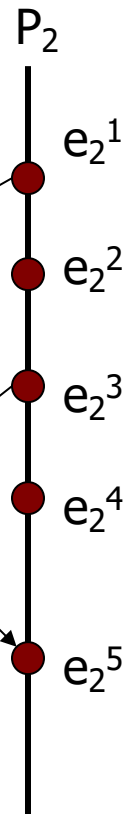


Consistent run

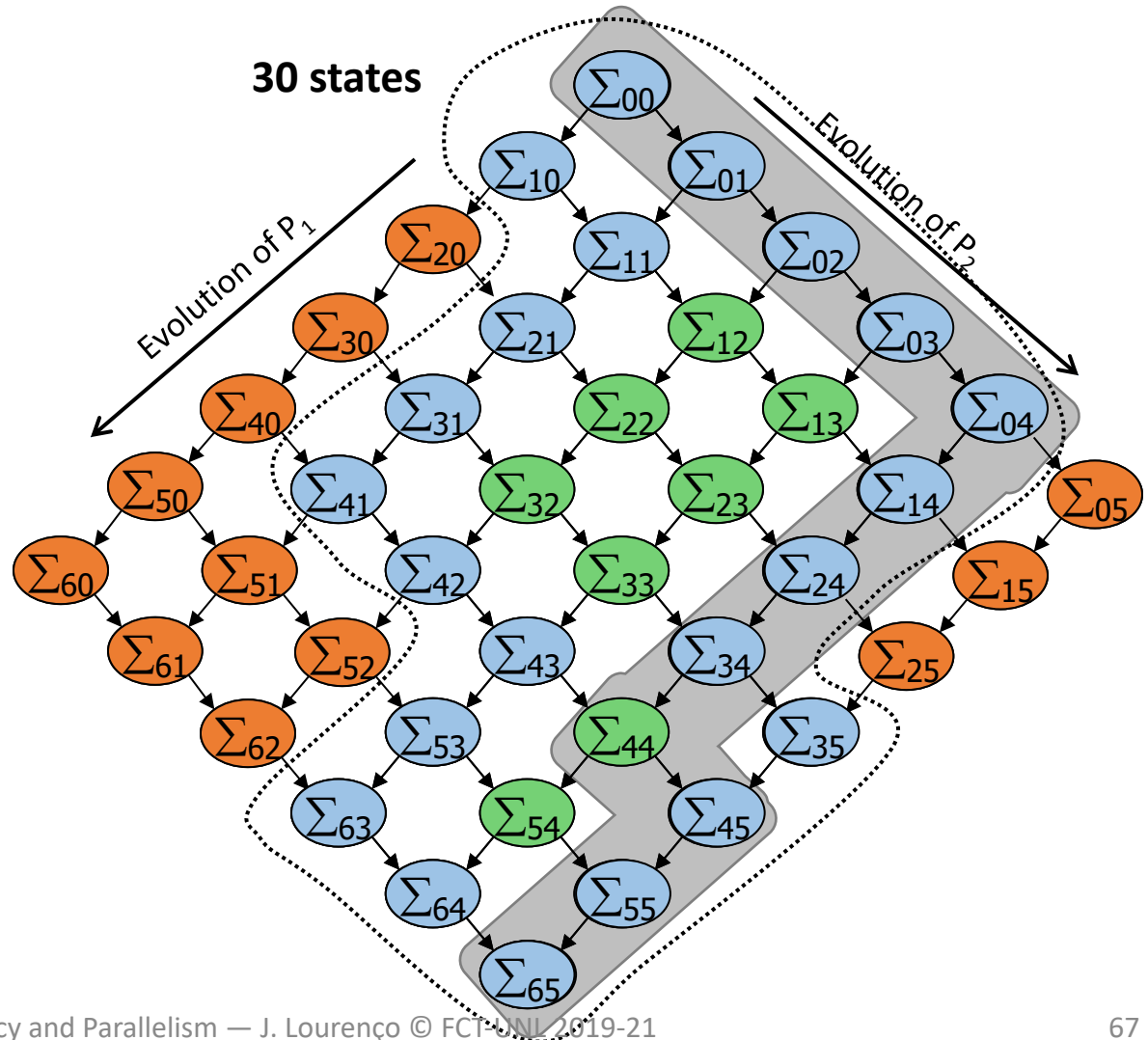
7 states



6 states



30 states

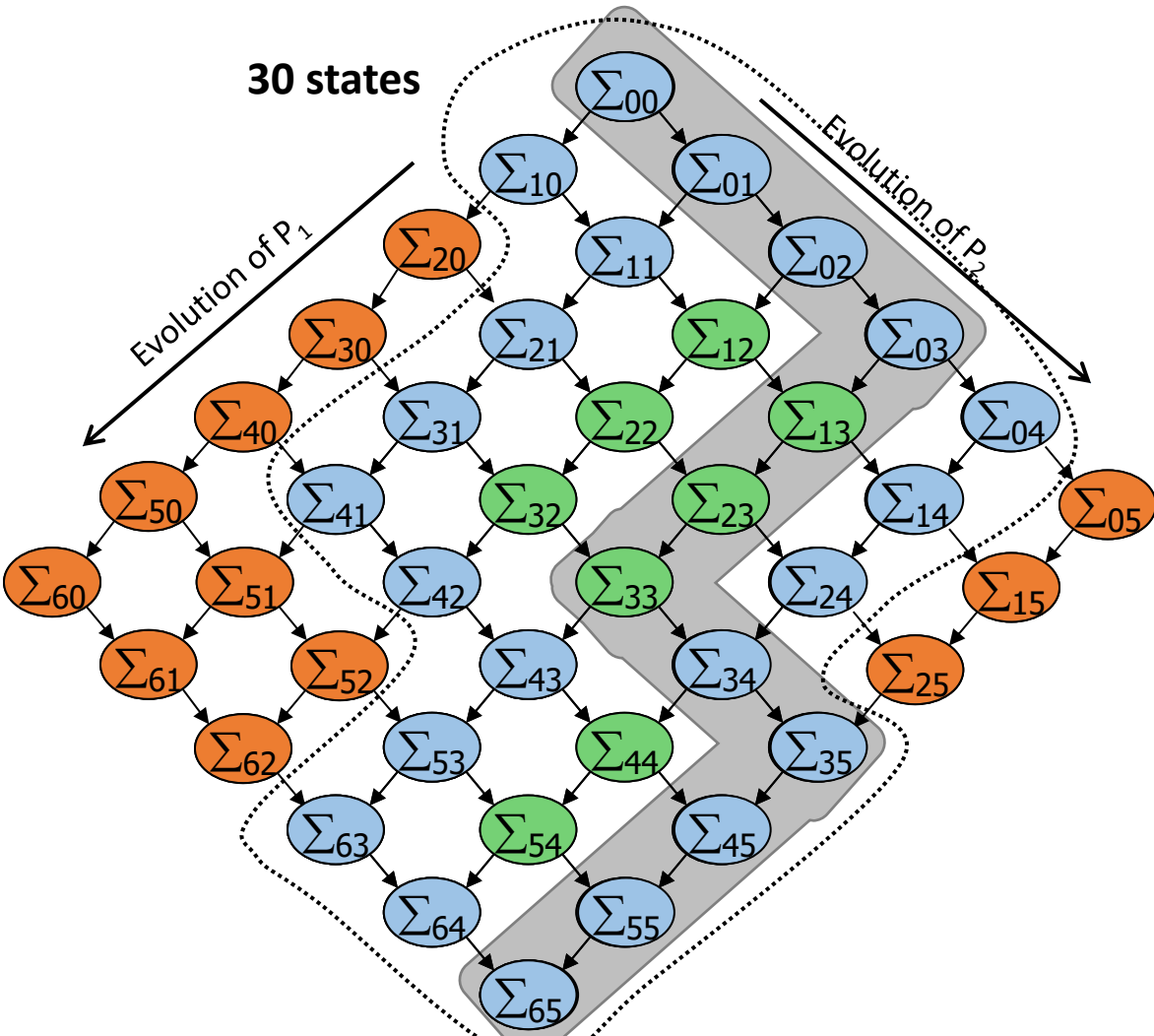
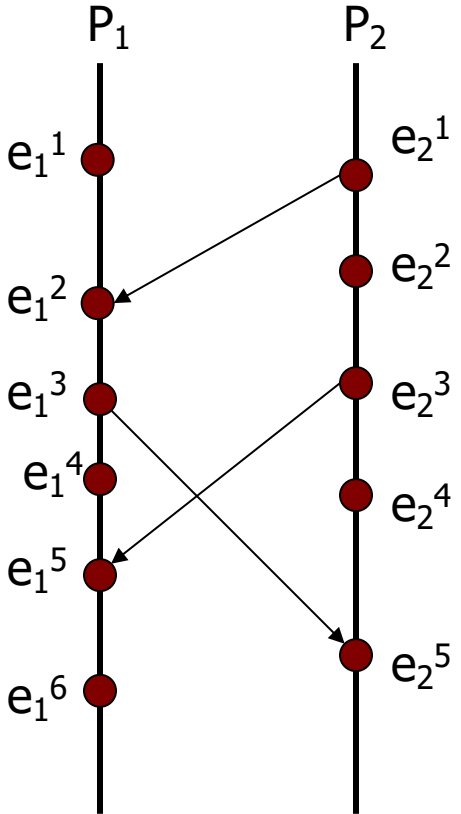


Consistent run

7 states

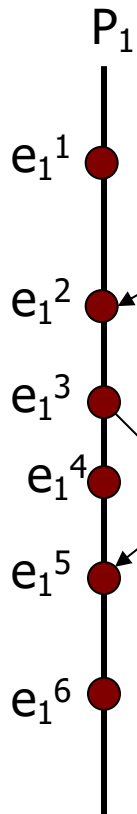
6 states

30 states

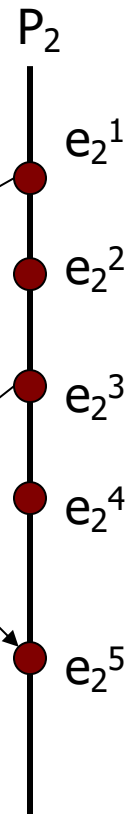


Consistent run

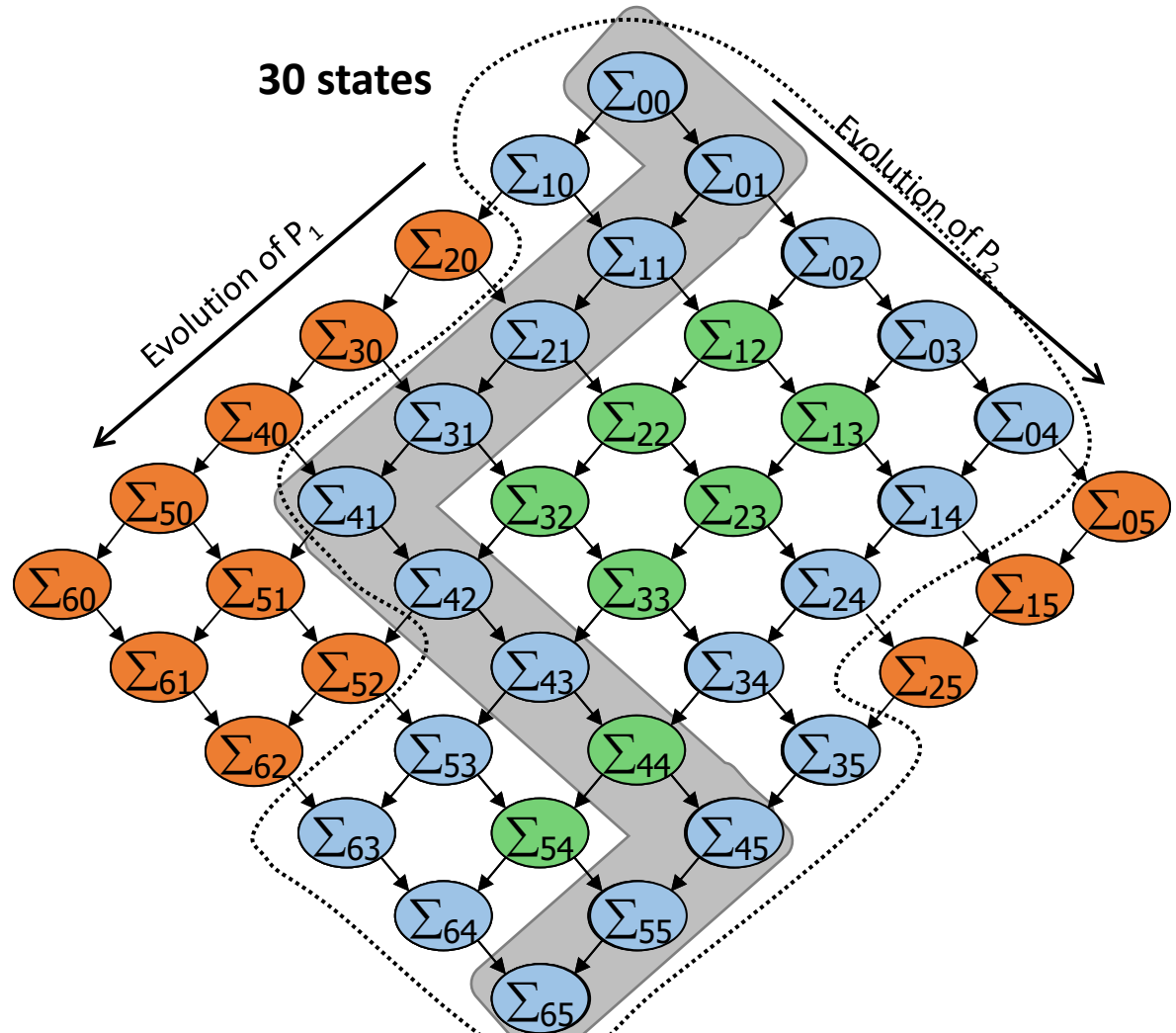
7 states



6 states



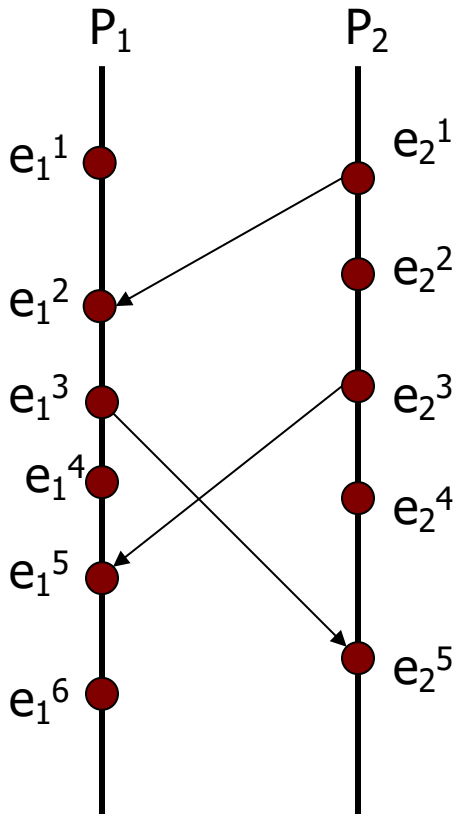
30 states



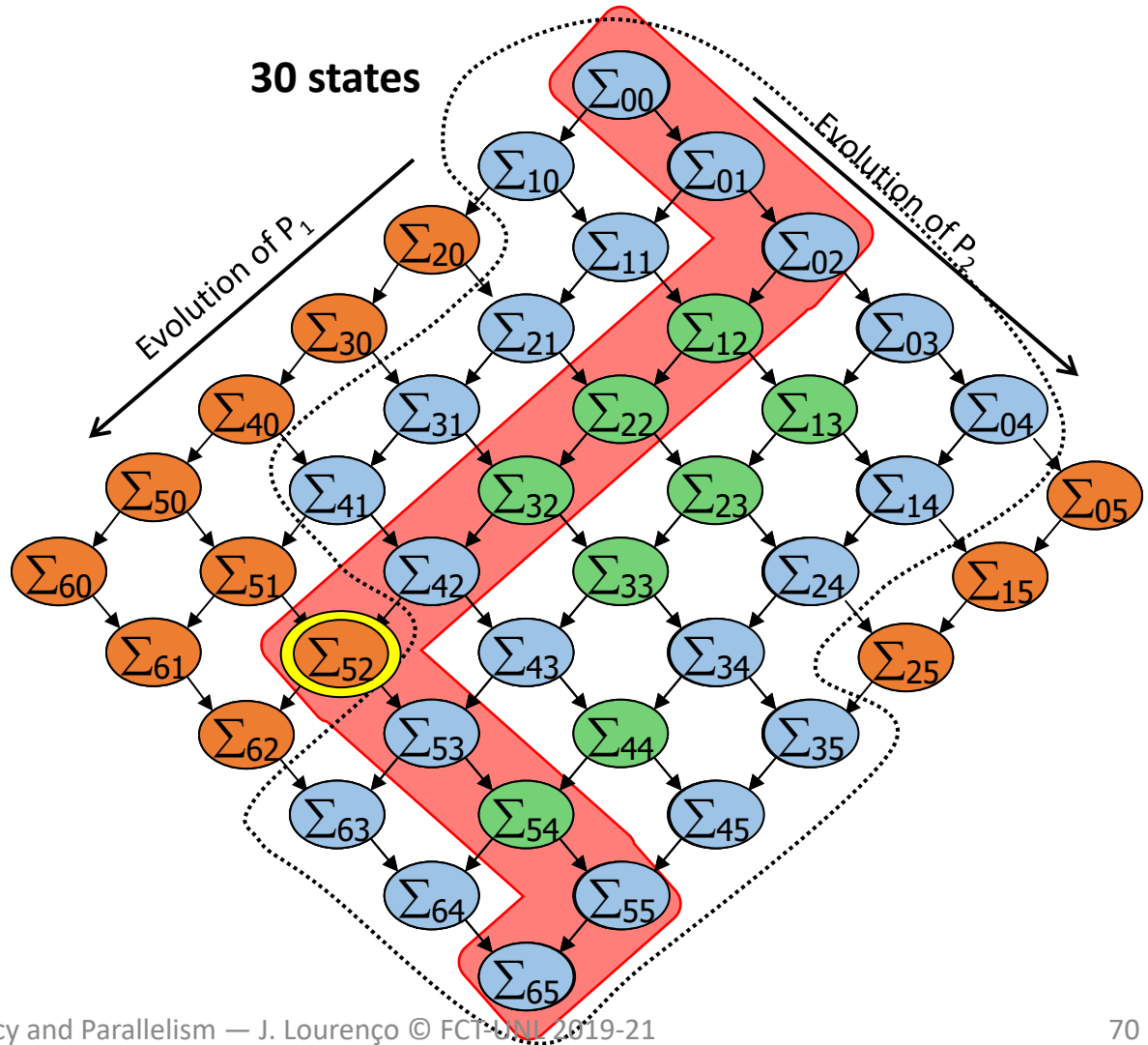
Inconsistent run – program error

7 states

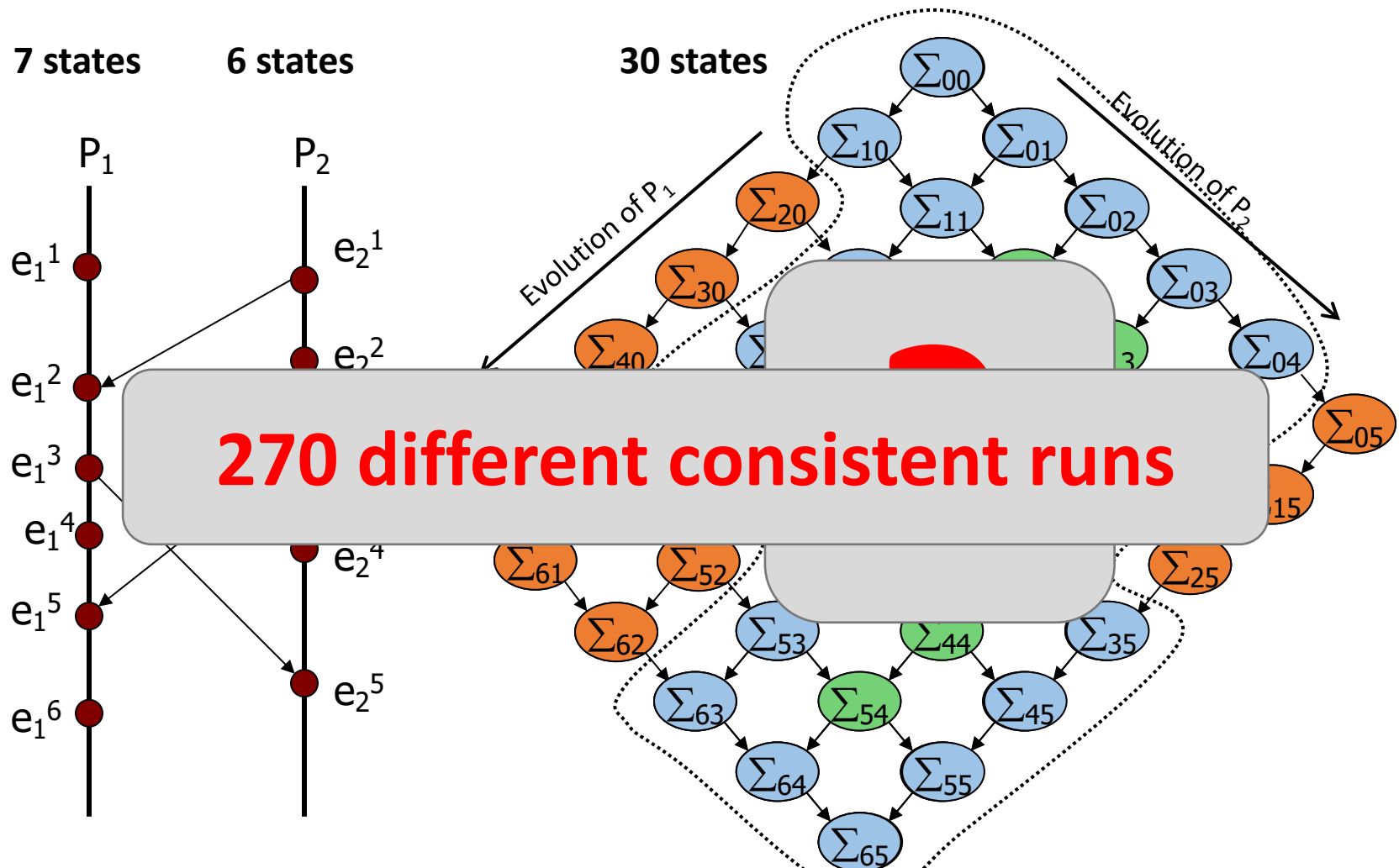
6 states



30 states



Consistent runs – How many?



The END
